Firm Investment in Energy Efficiency under Physical Energy Constraints

A Theoretical Exploration

Master's Thesis

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Abstract

This thesis explores whether long-run growth is feasible under a binding energy constraint and how firm investment in energy efficiency can support it. Building on the model by Bergmann & Kalkuhl (2025), energy intensity is modeled as an endogenous firm decision shaped by prices and investment costs. Simulations in MATLAB show that energy scarcity leads to rising prices, which incentivize firms to invest in efficiency. This allows growth to continue temporarily, but diminishing returns eventually lead to stagnation. While all cost functions support some efficiency gain, the long-run outcome depends on how steeply marginal investment costs increase. Moderate subsidies can extend the growth phase, but only to a limited extent. These findings highlight the need to align firm incentives with macro-level sustainability under physical limits.

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1. Introduction

Have you ever wondered how economic growth can continue in a world with limited energy? This is not just a theoretical question, but a pressing real-world challenge. As the global climate crisis intensifies, and fossil fuel reserves decline, societies face a fundamental challenge, namely, how to sustain development without surpassing the planet's physical limits. The concept of energy is central to this challenge, which is a resource so critical that, as Kümmel (2011) states, "nothing happens in the world without energy conversion and entropy production." Economic growth has historically always gone hand in hand with rising energy consumption (King, 2021; Smil, 2016). This creates a tension, because while governments aim to expand prosperity, they must do so under growing pressure to reduce emissions and limit energy use. After all, the shift to cleaner energy is not just about swapping one fuel for another, it is about preventing dangerous climate change and ensuring a sustainable future (Smil, 2016).

As renewable energy sources like solar and wind enter the mix, they offer hope, but they are not limitless. These sources are constrained by the laws of physics: sunlight is only available during the day, and wind flows are variable and unpredictable. This means that, even in a fully decarbonized system, energy availability will face a physical ceiling (King, 2021; Smil, 2016). In the long run, a fixed physical limit becomes relevant, such as the finite amount of solar energy that reaches the Earth each year. Once fossil energy stocks are exhausted, societies will have to rely entirely on this flow limit. This insight is central in the model by Bergmann & Kalkuhl (2025), who argue that future economic growth must occur within this boundary, making declining energy intensity a necessity.

Moreover, as King (2021) notes, "despite the development and exploration of new energy sources, the consumption of each individual fuel type has increased for the past 200 years," indicating that new energy sources tend to complement rather than replace existing ones. This implies that the transition we are undergoing is not only to a different type of energy, but to a fundamentally different type of economy, one that must grow with less energy input. Energy cannot easily be substituted by labor or capital, because without energy, workers cannot perform their tasks, and machines or buildings become nonfunctional. Energy is therefore essential for making both labor and capital productive (Keen et al., 2019).

This realization has brought energy efficiency to the center of climate policy. Reducing the amount of energy used per unit of output, also called lowering energy intensity, is now widely seen as a key solution to decouple economic growth from emissions. According to the IPCC (2023), "there is a rapidly closing window of opportunity to secure a livable and sustainable future for all," and demand-side solutions such as energy efficiency offer some of the most effective tools. The International Energy Agency (2023) notes that energy efficiency improvements could account for half of all emission reductions needed by 2030, while also reducing global energy bills by up to a third.

Despite the potential, a paradox persists, because many firms do not invest in energy-saving technologies, even when those investments seem to be cost-effective. This phenomenon, also known as the energy efficiency paradox, has been observed across different sectors and countries. The paradox can be explained by factors like uncertainty, irreversibility, and strategic delay, understanding these factors are critical for designing effective policies (Soest & Bulte, 2001; Ockwell, 2008).

To address this challenge, economists have started to explore how growth can still be possible when energy is limited. In a recent contribution, Bergmann & Kalkuhl (2025) present a growth model in which energy use is physically constrained, and long-run economic expansion is only possible if energy intensity declines. However, in their framework, the decline in energy intensity happens exogenously, it assumes that energy intensity automatically lowers. This raises a key question: what if firms could choose their level of energy efficiency, based on prices, costs, and policies?

In that case, their choices could play a key role in shaping the future of both energy use and economic growth. In the face of growing climate risks and physical constraints, understanding how firms respond to energy incentives is more urgent than ever. This thesis offers a step toward that understanding, and a contribution to the design of sustainable growth models that work within the planet's boundaries. This research focuses on the conditions under which firms invest in energy efficiency, and whether their choices are enough to support long-run economic growth under energy constraints. The research question guiding this thesis therefore is: *"What determines whether firms continue to invest in energy efficiency, and does this make it possible to sustain long-term economic growth when energy use is physically constrained?"*

To answer this question, a stylized theoretical model is developed in which firms strategically decide how much to invest in lowering their energy intensity, defined as the amount of energy required per unit of output. These decisions are based on a trade-off between investment costs and expected energy savings, and they are influenced by factors such as energy prices and policy incentives. The model takes the form of a profit-maximization framework in which energy intensity is an endogenous choice variable. It extends the work of Bergmann & Kalkuhl (2025) by making energy intensity responsive to firm behavior, rather than treating it as exogenously declining. This allows the model to explore how firm-level decisions interact with macro-level energy constraints, and to assess whether economic growth can continue in a world with limited energy.

Moreover, the model's structure makes it suitable for policy analysis. By simulating scenarios such as energy scarcity and investment subsidies, the thesis provides insights into how policy instruments can influence innovation and support a transition toward a more energy-efficient economy. This contributes to current debates on green growth and the role of firm behavior in meeting sustainability goals.

The findings highlight the crucial role of firm-level investment decisions in enabling sustainable growth under energy constraints. Without scarcity or policy, firms do not invest in energy efficiency, resulting in high energy intensity and unsustainable growth. A binding energy cap triggers price signals that drive efficiency investments, reducing energy intensity and enabling growth, until returns diminish and a zero-growth steady state emerges. While policy interventions like investment subsidies can extend the period of efficiency gains, they cannot fully prevent this.

This thesis is structured as follows. After this introductory chapter, Chapter 2 presents a literature review covering growth models, energy constraints, and firm incentives. Chapter 3 outlines the model structure and firm's decision-making framework. Chapter 4 present the different scenarios and results. Chapter 5 concludes the thesis, followed by a discussion in Chapter 6.

2. Theoretical background

This chapter aims to provide a detailed explanation of the variables identified in the previous chapter, drawing upon relevant literature on the associated concepts. The chapter starts by examining how economic growth interacts with environmental limits. The subsequent section discusses energy use and the physical constraints of it. The third section focuses on energy efficiency and firm behavior. The final section explores the role of policy.

2.1. Economic growth and environmental limits

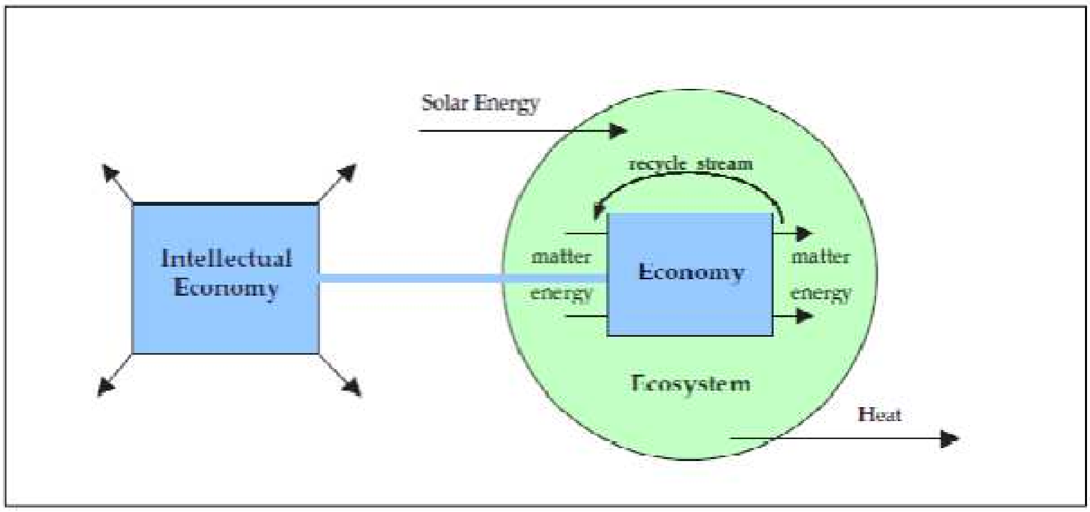
It has been long recognized that continuous growth in population, production, and consumption cannot be sustained indefinitely within a finite ecological system. As Hepburn and Bowen (2012) note, efforts to protect the environment will inevitably curb production growth and may even reduce national income levels. This outcome is a logical consequence of the fact that all economic activity ultimately depends on physical inputs from a finite environment. These concerns about environmental limits also resonate in the economic growth literature.

Early growth models, such as the Solow (1956) model, attributed long-run growth to exogenous technological progress and largely ignored environmental constraints. In response, Arrow (1962) introduced endogenous learning mechanisms, paving the way for models in which knowledge accumulation and innovation drive growth. Building on this, Romer (1990) and Aghion & Howitt (1992) emphasized how firms invest in R&D and innovation to sustain growth, even under diminishing physical returns. This literature indicates that technological progress is not entirely exogenous, but shaped by firms' strategic investment decisions, particularly when environmental constraints are present.

Considering these constraints, Hepburn and Bowen argue that sustained economic growth requires a structural shift from material to knowledge-intensive production. Rather than relying on ever-increasing material throughput, long-term prosperity must be built on innovation, services, and intellectual output forms of value creation that are not bounded by the physical ecosystem. Their concept of "green growth" proposes that firms, guided by policy incentives and technological opportunities, can reduce their material and energy intensity, and thus decouple economic activity from environmental degradation.

The distinction between a material and an intellectual economy is crucial in this context. As shown in Figure 1, the material economy is physically bounded by the ecosystem, it relies on flows of matter and energy and generates emissions and waste. In contrast, the intellectual economy, based on knowledge creation, can expand with far less environmental impact. Hepburn and Bowen emphasize that it is this kind of structural transition, not just marginal improvements, that will determine whether growth remains compatible with ecological sustainability.

Figure 1: The material economy is bounded by the ecosystem; the intellectual economy is not (Hepburn & Bowen, 2012)



While this conceptual distinction is important, Nordhaus (1992) provides one of the earliest attempts to quantify the "environmental drag" on economic growth due to physical resource constraints and pollution. His estimates suggest that limitations on non-renewable resources, such as energy and land, may reduce global output by up to 19.4% by 2050. This illustrates a tangible, though currently moderate, drag on the material economy, especially when energy inputs cannot be easily substituted.

These findings support the framework of Hepburn and Bowen (2012). They argue that the material economy is fundamentally constrained by biophysical limits. In contrast, the intellectual economy, which is driven by knowledge and innovation, is not constrained in the same way. From this perspective, long-term sustainability depends on a structural shift away from resource-intensive production toward knowledge-based value creation.

This shift is also reflected in the endogenous growth literature. Early models treated technological progress as exogenous. Later work by Romer (1986, 1990) and Aghion & Howitt (1992) emphasized how firm-level R&D and innovation can sustain growth in the face of diminishing physical returns. Firms do not passively adopt new technologies, but they actively invest to create them. Ulku (2004) further supports this view by linking innovation directly to R&D inputs such as human capital and existing knowledge.

Traditional economic models often overlook the realities of energy constraints. Growth frameworks rooted in the Solow model focus on capital accumulation, labor, and total factor productivity, while treating energy as a minor intermediate input. Bergmann and Kalkuhl (2025) model energy and machines as strict complements using a Leontief production function. This setup explicitly reflects thermodynamic constraints, where energy cannot simply be substituted by capital.

However, their model treats the decline in energy intensity as exogenous, which simplifies the complex reality of firm-level investment decisions. While their approach captures the important role of energy in economic output highlighted by ecological economics literature emphasizing energy's disproportionately large impact (Kümmel, 2011; Ayres & Warr, 2005), it overlooks how energy efficiency improvements arise from strategic firm behavior. Endogenizing energy intensity decline could provide deeper insights into how energy use and productivity interact under physical constraints.

Factor decomposition studies support this view, showing that energy contributes far more to productivity than its cost share suggests. Among all ecological limits to growth, energy plays a particularly central role. The next section zooms in on this constraint and explores the physical and economic limits to energy use in more detail.

2.2. Energy use and physical limits

In addition to economic considerations, physical and thermodynamic constraints impose further limits on energy use. The second law of thermodynamics holds that all energy conversions produce entropy, and thus that the efficiency of energy use cannot increase indefinitely (Ockwell, 2008; Kümmel, 2011). As energy systems approach their physical limits, further gains in efficiency become progressively more difficult and costly to achieve. Bergmann and Kalkuhl (2025) also acknowledge this reality, noting that once the technical potential of a given product or process is exhausted, future efficiency gains must come from the invention of entirely new varieties.

These physical limits are compounded by constraints on energy supply. Fossil fuels are both finite and highly polluting. Although renewable sources such as solar and wind offer a cleaner alternative, they are inherently limited by natural variability. Because solar radiation is only available during daylight hours, and wind flows are intermittent. As King (2021) and Smil (2016) emphasize, all historical energy transitions have been slow and incremental. New energy sources typically complement rather than fully replace existing ones. Despite substantial investments and policy efforts, fossil fuels still account for over 85% of the world's primary energy supply. This slow transition makes it unlikely that renewable energy alone will be able to support continued economic growth at current energy intensities.

Bergmann and Kalkuhl's (2025) model bring this issue into sharper focus. While their framework highlights that growth is only sustainable if energy intensity declines continuously, it does not explain how such a decline arises. By treating energy intensity as exogenous, the model misses an important opportunity to understand how firm behavior, driven by energy prices, policy, and innovation costs, could endogenously shape the future path of energy use and economic growth. This thesis builds on this gap by developing a model in which energy intensity is an endogenous decision variable, allowing for a more realistic representation of how firms respond to energy prices and policy incentives

2.3. Energy efficiency

Energy efficiency is widely recognized as a key strategy for reducing energy consumption and mitigating climate change. As Freire-Gonzalez (2015) observes, many national climate strategies rely heavily on efficiency improvements to lower greenhouse gas emissions. The underlying assumption is straightforward: using less energy per unit of output reduces total energy demand and, consequently, emissions. However, the practical realization of this potential depends critically on the behavior of firms and here, theory and practice often diverge.

Despite the apparent cost-effectiveness of many energy-saving technologies, firms often do not adopt them. This puzzling observation is known as the energy efficiency paradox. Jaffe and Stavins (1994) were among the first to systematically describe a persistent gap between the predicted and actual adoption of such technologies. They attributed this gap to a range of market failures and organizational barriers that prevent firms from behaving as standard economic models would expect.

Later research introduced a more dynamic explanation. Van Soest and Bulte (2001) emphasize the role of uncertainty and irreversibility in shaping firm investment behavior. They show that traditional models underestimate the value of waiting, that is, the strategic benefit of delaying investment in energy efficiency until more information becomes available or better technologies emerge. When investments are irreversible, as is often the case with energy technologies, firms face a real risk of locking themselves into suboptimal solutions. In this context, delaying action can be a rational strategy, not inefficient. This logic reframes energy efficiency as a strategic decision made under uncertainty, rather than a default outcome. Firms do not simply adopt efficient technologies when they appear cost-effective on paper; they weigh risks, anticipate future developments, and act accordingly.

2.4. Policy and technology choice

In recent years, the concept of directed technical change has become central to debates on sustainable growth. Acemoglu et al. (2012) provide a foundational framework, showing that the direction of innovation, whether toward clean or dirty technologies, is not exogenously determined but heavily influenced by policy. Instruments such as carbon taxes and subsidies do more than internalize externalities, they actively shape firms' investment choices and the pace of technological change.

This policy-driven view of innovation is echoed in the work of Hart (2004) and André & Smulders (2014), who argue that public intervention plays a key role in steering R&D toward socially desirable outcomes, such as energy efficiency. These insights challenge the common assumption in macroeconomic models that technological progress is an autonomous, market-driven process. Instead, innovation is responsive to incentive structures, and these can be designed.

Empirical evidence supports this policy-oriented perspective. Popp (2002), for example, finds that both the stringency and predictability of environmental regulation significantly affect the direction of innovation, as reflected in energy-related patenting activity. These findings reinforce the view that firms respond rationally to changing price signals, policy frameworks, and expected future costs, especially when it comes to energy.

However, unrealistic expectations about the pace and nature of energy transitions persist. As Smil (2016) warns, the rapid progress seen in sectors like electronics should not be compared to the much slower and more complex shift in the global energy system. While some improvements are possible through better technologies, the energy system is deeply tied to material infrastructure, making its transformation much more gradual.

Within this context, Mahmood and Ahmad (2018) offer further nuance. Their work shows that energy intensity tends to decline with economic growth. Not just due to autonomous technical progress, but because of structural economic shifts and gradual adoption of less energy-intensive technologies. This underscores the importance of modeling technology choice at the firm level, since micro-level decisions collectively determine macro-level outcomes.

Despite these insights, many macroeconomic growth models still treat energy intensity (ε) as exogenous, limiting their policy relevance. Bergmann and Kalkuhl (2025) also adopt this assumption, even though their model shows that sustained growth requires a declining energy intensity. They suggest future research should endogenize this choice, a step this thesis takes.

While real-world firms vary widely in their energy use and responsiveness to policy, this thesis adopts a representative firm framework to isolate the core economic mechanisms.

Although this abstraction omits sectoral or geographic differences, it remains grounded in empirical findings, which show that policy responsiveness differs significantly by firm size, sector, and technological capability (Mahmood & Ahmad, 2018; Popp, 2002).

Whereas the literature reviewed in this chapter presents a diverse and complex picture of energy transitions and innovation behavior, the model developed in this thesis adopts a more abstract and stylized approach. It does not attempt to capture sectoral or firm-level heterogeneity, nor does it include uncertainty or institutional complexity. Instead, the goal is to isolate key theoretical mechanisms, in particular, how firm investment decisions in energy efficiency interact with macro-level energy constraints.

This simplification allows the model to serve as a conceptual tool that links ideas from endogenous growth theory, ecological economics, and directed technical change. By focusing on the interaction between firm-level decisions and physical resource limits, the model helps clarify under what conditions sustainable growth might or might not be achievable

3. Model design

This chapter presents the structure of the theoretical model developed in this thesis. It investigates how firm-level decisions regarding energy efficiency interact with macro-level constraints on total energy use. The model builds on Bergmann & Kalkuhl (2025) but introduces a key modification by endogenizing energy intensity as a firm-level decision. The core question is whether long-run growth remains feasible when total energy use is subject to a fixed energy cap, and whether firms voluntarily continue to invest in efficiency under different market and policy conditions.

The economy consists of a range of differentiated products, each produced by a firm that enters sequentially. Upon entry, a firm chooses how much to invest in energy efficiency, which determines its energy intensity. This choice affects both production costs and aggregate energy demand. As the number of product varieties expands, total output increases, but total energy use is capped. This setup creates a dynamic trade-off between firm-level efficiency and economy-wide growth.

3.1. Overview and assumptions

This thesis extends the model of Bergmann & Kalkuhl (2025), by making energy intensity endogenous. Firms choose their level of energy efficiency based on investment costs and energy prices. This modification allows us to explore whether long run growth remains feasible under a fixed energy ceiling. The model is implemented in MATLAB. All figures presented in this thesis are generated using MATLAB scripts, with each script corresponds to a specific scenario. An overview of all scripts is provided in Appendix – MATLAB Scripts. The economy consists of three types of agents:

- Households: they consume the final output and provide labor.
- Producing companies: Each firm produces a unique product variety. However, since all firms are assumed to be structurally identical, they face the same cost function and optimization problem. Therefore, energy intensity is treated as identical across firms, and we drop the firm index j for simplicity.
- R&D companies: they innovate and create new product varieties N(t), expanding economy over time.

This setup reflects the product variety model of growth, in which expanding the number of goods drives long-run output growth.

The model relies on the following assumptions:

- Homogenous firms: all producing firms face the same cost structure and optimization problem.
- Perfect competition on product markets: firms are price takers in output market, but innovation involves imperfect innovation.
- Perfect foresight and full information: all agents know future prices and constraints.

- No uncertainty or information problems: firms can invest freely in energy efficiency without friction.
- A binding physical energy limit: total energy use cannot exceed a fixed ceiling, which represents the long-run renewable energy potential.

These assumptions reflect standard simplification used in endogenous growth theory and are consistent with the structure used by Bergmann & Kalkuhl (2025).

3.2. Firm behavior and profit maximization

In this model, each firm produces a unique product variety and decides how much to invest in energy efficiency. The key decision variable is the level of investment y, which determines the firm's energy intensity. Lower energy intensity reduces energy use per unit of output but comes at a cost. Firms maximize profits by choosing their investment y, with the profit function given by:

$$\pi_j(y) = output - costs$$

The profit function from the research of Bergmann & Kalkuhl (2025) is shown below:

$$\pi_j(t) = (1-a)aAL(\frac{A\alpha^2}{\varepsilon_j Q(t)+1})^{\frac{\alpha}{1-\alpha}} - c(y_j)$$

Where:

- α: output elasticity of capital and energy
- A: productivity parameter
- L: labor
- Q: energy price
- $\varepsilon_j(y)$: energy intensity, decreasing in investment y
- $c(y_j)$: cost of energy efficiency investment

In this model, y represents a firm's investment in energy efficiency, interpreted as the amount of labor input devoted to innovation. This could correspond to the number of researchers or engineers working on energy-saving technologies. Importantly, y is expressed in labor units and not in monetary terms. The cost functions are thus illustrative and reflect relative trade-offs rather than absolute financial values.

To find the optimal investment, firms take the first-order condition of the profit function. Since energy intensity is a function of y, this requires applying the chain rule:

$$\frac{d\pi_j}{dy_j} = \frac{1}{\rho} * \frac{d\pi_j}{d\varepsilon_j} * \frac{d\varepsilon_j}{dy_j} - \frac{dc}{dy_j}$$

For the functional forms used in this thesis, the derivative becomes:

$$\frac{d\pi_j}{dy_j} = \frac{1}{\rho} * [\frac{\alpha}{1-\alpha} * \frac{Q}{\varepsilon_j Q + 1} * \pi_j \ * \beta\varepsilon_j] - \frac{dc}{dy_j}$$

The first term captures the marginal benefit of investing in energy efficiency, discounted over time by ρ. The second term captures the marginal cost of investing in energy efficiency. Setting this derivative equal to zero characterizes the firm's optimal investment, where marginal cost equals marginal benefit. Under the assumption of diminishing returns (a negative second derivative), this yields a local maximum. In practice, the optimal investment is computed numerically in MATLAB for each product variety.

3.3. Energy efficiency and cost functions

Firms reduce their energy intensity by investing a certain amount y. The relationship between investment and energy intensity is given by:

$$\varepsilon_j(y_j) = \varepsilon_{j-1} * e^{-\beta y_j}$$

This implies that higher investment reduces energy intensity exponentially. The parameter β determines how effective the investment is. A higher β means faster reductions in energy use per unit of output. The term ε_{j-1} reflects the initial energy intensity before investment. In this model, y represents labor input allocated to R&D for energy efficiency. It can be interpreted as the number of researchers or the scale of technical effort aimed at reducing energy use. The investment cost function in the main scenario is:

$$c(y_j) = \frac{\left[\alpha_c y_j + \frac{1}{2}\beta_c {y_j}^2\right]}{\varepsilon_{j-1}(y_j)} \text{ with } \frac{dc}{dy_j} = \frac{\alpha_c + \beta_c y_j}{\varepsilon_{j-1}(y_j)}$$

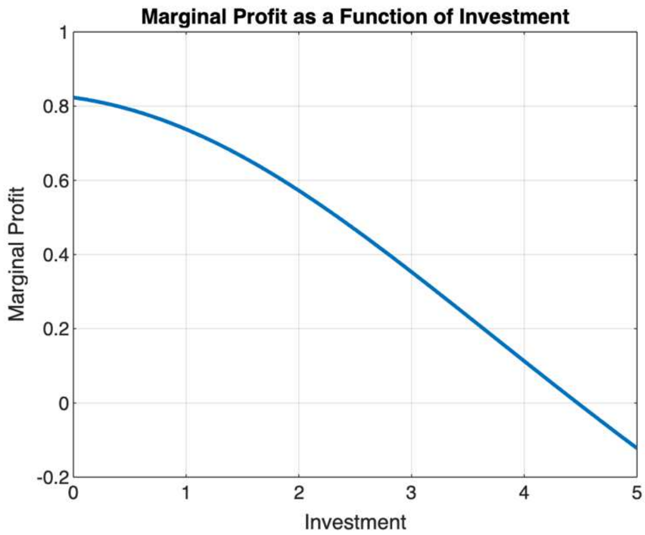
This specification implies that firms not only pay for how much they invest but also for how much efficiency improves relative to their starting point. Firms with higher initial energy intensity can improve more cheaply, making early-stage gains more accessible. The optimal investment is found by equating the marginal benefit of reduced energy costs with the marginal cost of investment, as derived in the previous section. The total derivative of profit with respect to y_j is:

$$\frac{d\pi_j}{dy_j} = \frac{1}{\rho} * [\frac{\alpha}{1-\alpha} * \frac{Q}{\varepsilon_j Q + 1} * \pi_j \ * \beta\varepsilon_j] - \frac{\alpha_c + \beta_c y_j}{\varepsilon_{j-1}(y_j)}$$

3.3.1. Marginal profit visualization

Figure 2 below illustrated how marginal profit varies with investment in energy efficiency. At low levels of investment, the marginal profit is positive, the benefits from improving energy efficiency outweigh the costs. As investment increases, marginal profit declines due to diminishing returns in energy savings, rising investment costs, and smaller additional gains. The curve eventually crosses zero, indicating the profit-maximizing level of investment. This optimum is found numerically in MATLAB by identifying where marginal profit equals zero.

Figure 2: Marginal profit as a function of investment



3.3.2. Different cost functions

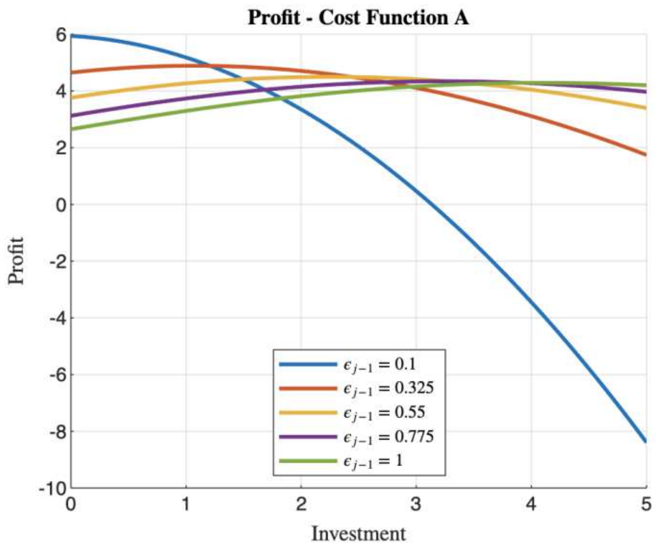
To explore how different economic assumptions about investment behavior influence firm decisions, this thesis compares five distinct cost functions. Each function reflects a different view on how energy-efficiency investments interact with a firm's initial energy intensity and its chosen investment level. For each cost function, there is a simulation and visualization how profit varies with investment for firms that differ in their starting energy intensity. This approach allows to isolate how the structure of the cost function shapes the firm's optimal choice.

The resulting graphs serve as an illustration of how firms' incentives shift under different cost structures, helping to understand under which conditions energy-efficiency investments are attractive.

Cost function A:

$$c(y_j) = \frac{\left[\alpha_c y_j + \frac{1}{2}\beta_c {y_j}^2\right]}{\varepsilon_{j-1}(y_j)}$$

Figure 3: profit as a function of investment, using cost function A

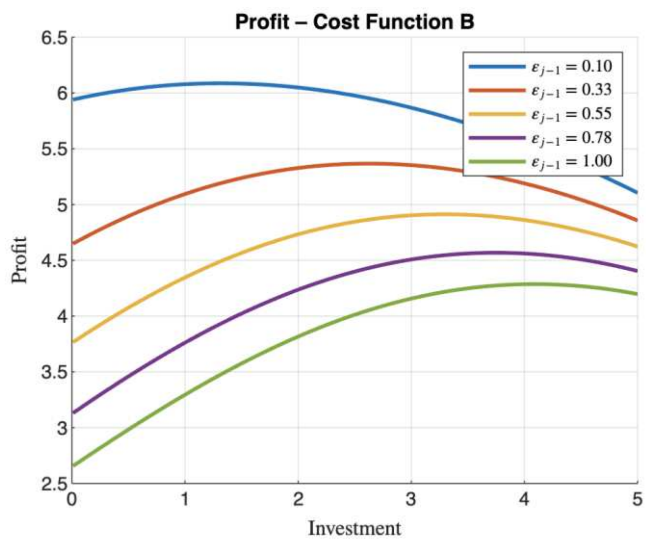


This cost function includes an exponential relationship between investment and energy efficiency. As firms invest more, energy intensity decrease, which makes costs rise faster due the shrinking denominator. Figure 3 shows that firms with lower initial energy intensity reach peak profit at lower investment levels, since further improvements become more costly. This illustrates how diminishing returns shape investment behavior, especially near the energy efficiency frontier.

Cost function B:

$$c(y_j) = \left[\alpha_c y_j + \frac{1}{2}\beta_c {y_j}^2\right]$$

Figure 4: profit as a function of investment, using cost function B

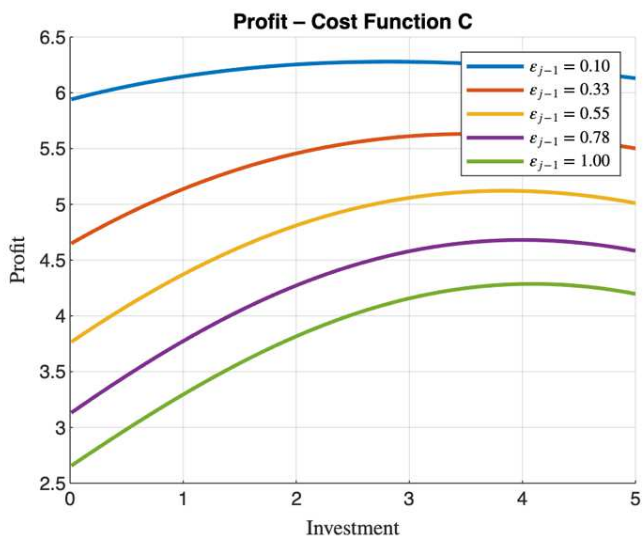


This cost function assumes that investment costs are purely a function of the investment level, and not of prior energy efficiency. It follows a standard quadratic form. While all firms face the same cost curve, their benefit differs. Firms with lower ε_{j-1} achieve higher output for the same investment, resulting in higher profits. As shown in Figure 4, these firms reach their peak profit earlier.

Cost function C:

$$c(y_j) = \left[\alpha_c y_j + \frac{1}{2}\beta_c {y_j}^2\right] * {\varepsilon_{j-1}}^{\gamma}$$

Figure 5: profit as a function of investment, using cost function C

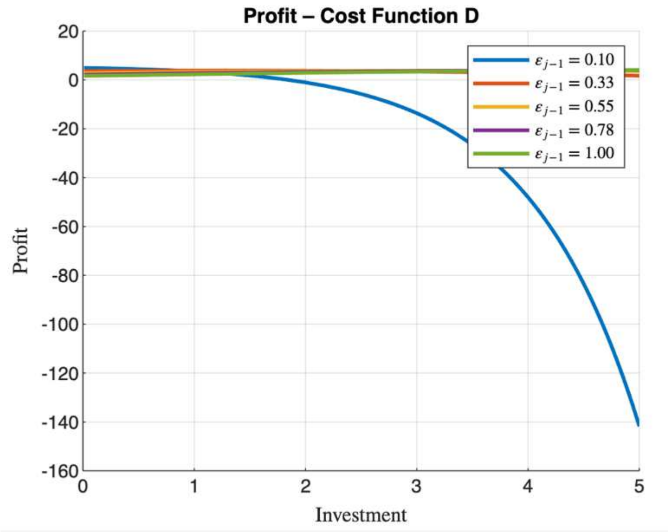


This cost function assumes that more energy-efficient firms face lower investment costs, as costs are scaled by energy intensity. The parameter γ determines how strongly efficiency affects costs, higher values increase this effect. As shown in Figure 5, profits are higher and increase faster for more efficient firms, since their investment is relatively cheaper. This setup captures the idea that "clean" firms are rewarded, either by market forces or policy, with lower costs for becoming even cleaner.

Cost function D:

$$c(y_j) = e^{\zeta y_j / \varepsilon_{j-1}}$$

Figure 6: profit as a function of investment, using cost function D

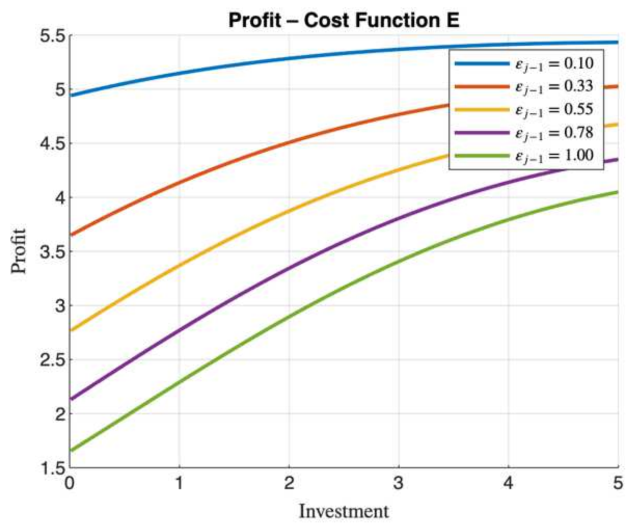


This cost function assumes that investment costs rise exponentially with effort, especially when firms are already energy efficient. This reflects the idea that it becomes increasingly difficult and expensive to improve efficiency near physical or technological limits. Figure 6 illustrates this clearly, because profits decline sharply as investment increases, particularly for efficient firms. Even with a relatively low $\zeta = 0.1$, only limited investment remains profitable.

Cost function E:

$$c(y_j) = e^{\zeta y_j * \varepsilon_{j-1}{}^{\gamma}}$$

Figure 7: profit as a function of investment, using cost function E



This cost function assumes that investment costs grow exponentially with effort, but less so for firms that are already energy efficient. As a results, efficiency becomes self-reinforcing, because the cleaner the firm is, the cheaper it becomes to improve further. Figure 7 shows that profit steadily rises with investment across all efficiency levels, especially for firms with low initial energy intensity. With parameter values $\zeta = 0.1$, and $\gamma = 0.5$, inefficient firms face stronger cost penalties, encouraging them to invest and catch up with cleaner competitors.

3.3.3. Parameter settings

In this thesis, the parameters are not formally calibrated to empirical data but are chosen to be illustrative and internally consistent. This approach is in line with Bergmann & Kalkuhl (2025), who also do not perform a formal calibration. The goal is to generate meaningful simulations that highlight model behavior rather than predict real-world outcomes. The parameters used are:

- α = 0.6
- A = 7.0
- L = 1
- Q = 0.86
- β = 0.4
- ε_0 = 1.0
- α_c = 0.05
- β_c = 0.1
- ζ = 0.1
- γ = 0.5

Most parameters are chosen to be theoretically plausible and to ensure consistent model behavior. The initial energy intensity denoted as ε_0, which represents the amount of energy required to produce one unit of economic output at the beginning of the simulation is calibrated using global data: in 2024, total electricity consumption in that year was approximately 1,080.1 TWh (IEA, 2024) and global GDP was $110,549.44 billion USD (Statista, 2024), implying and empirical energy intensity of 9.77 GWh per billion USD. For simplicity and internal consistency, the initial energy intensity is normalized to ε_0 = 1.

The energy price Q = 0.86 is used across simulations, as it corresponds to the endogenous price level at N = 100 varieties under cost function A.

To assess how sensitive the model is to the parameter β, which governs how strongly energy efficiency improves with investment, a sensitivity analysis was performed for values between 0.2 and 0.6. As Table 1 shows, increasing β leads to higher optimal investment, greater profits, and significantly lower energy intensity. However, beyond $\beta = 0.4$ returns start to diminish. This value is therefore used as the baseline, offering a balanced trade-off between investment effort and energy efficiency.

The parameter values are also chosen that in the absence of an energy constraint, the model exhibits sustained growth (Bergmann & Kalkuhl, 2025). Furthermore, growth continues if firms keep investing, since each new product leads to a reduction in energy intensity. Since each product variety corresponds to a new period, the figures in this thesis can be interpreted as dynamic developments over time.

Table 1: sensitivity analysis beta

Beta	y* (optimal)	Profit	Epsilon (y*)
0.2	3.1045	3.1638	0.5375
0.3	3.9416	3.7575	0.3065
0.4	4.0881	4.2867	0.1949
0.5	3.9830	4.6976	0.1365
0.6	3.8031	5.0103	0.1021

3.4. Macroeconomic framework

This section outlines how output, energy demand, and growth are linked in the model. It combines firm-level decisions with aggregate constraints and builds on the approach of Bergmann & Kalkuhl (2025). Each firm produces output using two essential inputs: capital and energy. The model assumes a Leontief production function, which implies that capital and energy are perfect complements.

Since all firms are assumed to be structurally identical, they face the same production function, optimization problem, and input costs. While firms are structurally identical in terms of preferences and technology, each new firm that enters chooses its own investment level, and if y > 0 it results in a lower energy intensity than previous firms. As such, energy intensity varies across firms. The energy use per firm is:

$$E_j = \varepsilon_j(t) * q_j$$

$$E_j = \varepsilon_{j-1} * e^{-\beta y_j} * q_j$$

While the model simulates growth in product variety, it does not contain an explicit time index such as t = 0, 1, 2, .. Instead, growth is represented as a sequence of firm-level innovations, where each new variety j corresponds to an additional firm. This approach allows to approximate long-run outcomes, such as whether the system reaches a growth path or converges to a steady state. Hence, when referring to growth in N(t), that is describing the qualitative expansion of variety over the sequence of firms, not a time-based rate of change.

Aggregate energy use is the sum of all firm-level energy demands:

$$E(t) = \int_0^{N(t)} \varepsilon_j(t)dj$$

In a simplified setting with identical firm this reduces to:

$$E(t) = N(t) * \varepsilon(t)$$

The model assumes that aggregate energy use is subject to a physical ceiling, for example like mentioned in the introduction, the amount of energy that can be sustainably harvested from solar power. This constraint is written as:

$$E(t) \leq \bar{E}$$

The energy constraint operates through the following complementarity condition:

- If $E(t) < \bar{E}$, then Q = 0.
- If $E(t) = \bar{E}$, then Q > 0.

This mechanism ensures that energy becomes costly only when it is scarce, creating an economic incentive for firms to invest in energy efficiency.

Growth in the model is represented by an increase in $N(t)$. However, this growth is not unconstrained: once aggregate energy use hits the ceiling, further expansion in $N(t)$ is only possible if energy intensity reduces. This introduces an indirect dependency between product variety and firm-level efficiency: the more efficient firms are, the more varieties the economy can support. In other words, growth is only sustainable under the energy cap if firms become increasingly energy efficient. This aligns with the structure in Bergmann & Kalkuhl (2025):

$$E(t) = \sum_{j=1}^{N(t)} \varepsilon_j$$

Finally, since each firm produces one unique variety, total output grows one on one with $N(t)$. Therefore, the growth rate of economic output equals the growth rate of variety:

$$g_Q = \frac{\dot{Q}}{Q} = \frac{\dot{N}}{N} = g_N$$

If firms keep on investing in efficiency, $\varepsilon(t)$ declines and the economy can continue to grow without breaching the energy ceiling.

3.5. Equilibrium conditions

This section explains how the different components of the model interact and how equilibrium is defined at both the micro and macro level. The goal is to understand how firms' investment choices in energy efficiency interact with aggregate energy constraints and prices.

At the firm level, equilibrium is reached when each firm chooses the optimal level of energy efficiency such that profit is maximized:

$$Max\, \pi(y) = output\ (y) - cost\ (y)$$

Subject to:

$$\varepsilon_j(y) = \varepsilon_{j-1} * e^{-\beta y}$$

Solving this problem yield the optimal investment and the correspond optimal energy intensity. This value balances the gains from lower energy use against the cost of efficiency improvements.

At the aggregate level, total energy demand must not exceed the fixed physical limit $\bar{E}$, representing the renewable energy ceiling. This led to the following energy constraint:

$$E(t) = \int_0^{N(t)} \varepsilon_j * x_j(t) dj = \bar{E}$$

At the macro level, equilibrium requires that:

- Total energy use $E(t)$ does not exceed the physical energy ceiling.
- The complementary condition holds:
  If $E(t) < \bar{E}$, then Q = 0 (no scarcity)
  If $E(t) = \bar{E}$, then Q > 0 (scarcity price emerges)

To determine whether energy demand exceeds the cap, we compute the aggregate energy use based on firms' choices of ε and resulting output. The aggregate intermediate output is given by Bergmann & Kalkuhl (2025):

$$X(t) = \int_0^{N(t)} x_j(t) dj = \int_0^{N(t)} L\left(\frac{Aa^2}{\varepsilon_j Q(t) + 1}\right)^{\frac{1}{1-\alpha}}$$

Energy use per firm is:

$$E_j(t) = \varepsilon_j * x_j(t)$$

Total energy demand becomes:

$$E(t) = \int_0^{N(t)} \varepsilon_j * x_j(Q) dj$$

To satisfy the cap that E(t) is equal to $\bar{E}$, the model solves for the equilibrium price such that $E(Q) - \bar{E} = 0$.

3.6. Scenarios and simulation design

To simulate how firms behave under different constraints, the model numerically determines the energy price Q^*, a such that total energy demand equals the fixed cap. This is done using a root-finding algorithm (*fsolve*) in MATLAB.

The procedure begins by fixing the number of firms N. Given this number, an initial guess is made for the energy price Q. Based on this value, the model computes, for each firm j = 1, 2, …., N, the optimal investment level, which in turn determines the firm's energy intensity. Using the resulting energy and output, the energy consumptions of each firm is calculated. Aggregating the energy use across all firms gives the total energy demand. If the total energy use deviates from the physical energy ceiling, the energy price is adjusted, and the entire procedure is repeated. This iterative process continues until total energy demand converges to the energy constraint.

To explore how equilibrium outcomes scale with market size, the procedure can be repeated for increasing values of N, starting from N = 1 and incrementing up to a maximum number of firms (for example, N = 300). At each step, the model solves for the corresponding equilibrium energy price and firms' behavior. This approach allows the analysis to capture how energy scarcity and investment incentives evolve as the number of varieties in the economy expands.

In addition to determining Q, custom MATLAB functions are used to simulate how firms respond to different environments. These functions calculate, for example, the optimal level of investment for each variety, the resulting energy intensity and profit levels. Each scenario is modeled using a separate script, and figures are directly linked to these simulation results.

As mentioned before the goal is not to trace real-time dynamics, but to simulate long-run outcomes under different constraints and policy settings. So, it compares the following scenarios:

Scenario 1: Free market (no scarcity, no policy).

- Energy is abundant, so Q = 0.
- Firms have no incentive to invest.

- Goal: check whether voluntary investment is sufficient to keep energy use below $\bar{E}$.

Scenario 2: Energy scarcity (binding energy cap).

- Energy demand hits the physical limit, so Q > 0.
- Firms face higher energy prices and therefore stronger incentives to reduce ε.
- Goal: assess whether scarcity alone is enough to induce sufficient efficiency improvements.

Scenario 3: Policy intervention (subsidy).

- A subsidy reduces the cost of investment $c(y)$, a subsidy is implemented by multiplying this cost function by $(1-\sigma)$.
- Firms face a lower marginal cost of efficiency improvement.
- Goal: evaluate whether policy can accelerate efficiency investments and enable sustainable growth under the energy constraint.

4. Results

In the previous chapter, the theoretical model is explained. This chapter applies the model to simulate different scenarios and functions. And therefore, evaluates how energy efficiency, total energy use, and profits respond under these conditions.

4.1. Baseline scenario: no scarcity, no policy.

This section presents the results of the free-market scenario when there is no energy constraint and no policy intervention. It explores whether firms voluntarily invest enough in energy efficiency when energy is abundant and the price is zero (Q = 0) (Bergmann & Kalkuhl, 2025).

Figure 8: baseline scenario with Q=0, given cost functions A, B, C, D and E

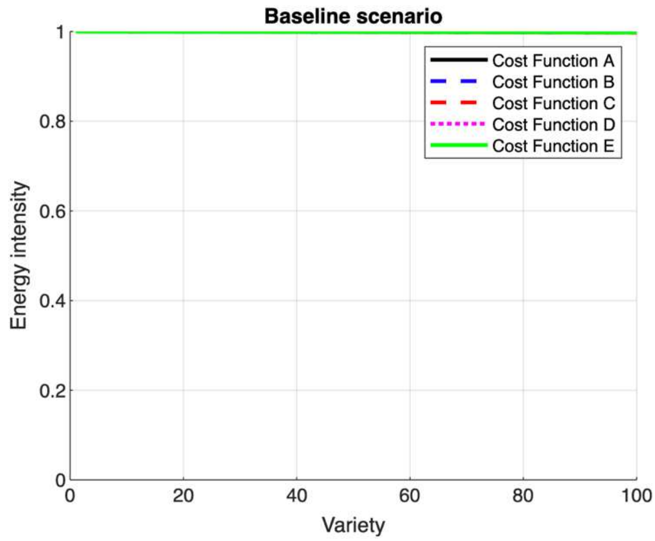


Figure 8 shows the evolution of energy intensity across product varieties when there is no energy constraint, and the energy price is zero (Q=0). The model is parametrized such that growth is possible even in the absence of an energy constraint, if firms invest in efficiency and energy intensity declines over time. However, as shown in figure 7, when energy is free ($Q = 0$), firms do not invest, and energy intensity remains constant at 1 across all product varieties.

This outcome holds across all cost functions (A–E), confirming that without external incentives such as an energy price or physical energy cap, firms do not voluntarily improve efficiency. This baseline scenario therefore provides a useful benchmark. It illustrates firm behavior in a free market without scarcity or policy, where energy use per product remains high. As a result, under a fixed energy ceiling ($\bar{E} = 150$), the number of product varieties that can be supported is severely limited. This underlines the importance of policy or scarcity in enabling sustainable long-run growth.

4.2. Scarcity scenario: binding energy constraint.

This section presents the results when a binding energy ceiling ($\bar{E} = 150$) is introduced. Unlike the baseline scenario, energy is now scarce once the cap is reached and the energy price becomes positive ($Q > 0$), which creates incentives for firms to invest in efficiency. The energy price is determined endogenously to ensure that total energy use remains within the limit. The key question is whether firms adapt their behavior sufficiently to maintain growth.

Figure 9 shows how firms respond under cost function A when the number of varieties is fixed at $N = 300$. In the beginning, when energy demand is still below the cap, the energy price remains zero (blue dashed line). However, once aggregate energy use reaches the ceiling, Q becomes positive and starts rising with each additional variety. As a result, firms begin to invest in energy efficiency. This leads to a continued decline in energy intensity (solid black line).

Figure 9: energy intensity and energy price per variety

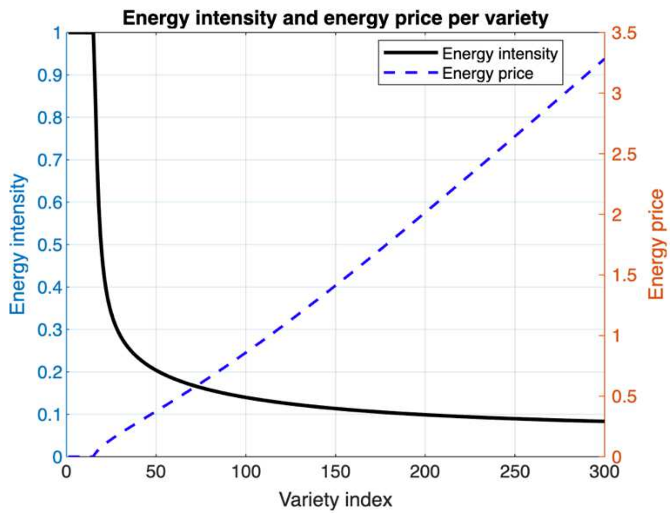


Table 2 illustrates how the energy constraint shapes equilibrium outcomes as the number of product varieties increases. For N = 100, an energy price of Q = 0.86 suffices to keep total energy use at the cap of 150. At this price, firms reduce their energy intensity 0.14, which allows production to remain within the constraint without significantly limiting variety. As N rises to 200 and 300, total output increases, which would exceed the energy cap unless energy use per product declines further. The model responds with higher equilibrium prices of Q = 2.01 for N = 200 and Q = 3.28 for N = 300, which incentivize firms to make stronger efficiency improvements. This results in progressively lower energy intensities, dropping to $\varepsilon(t)$ = 0.099 and $\varepsilon(t)$ = 0.083, respectively.

Table 2: equilibrium price, epsilon and total energy use under constraint

N	Cost function	Q	$\varepsilon(t)$	Total energy
100	A	0.8593	0.1399	150.000
200	A	2.0148	0.0989	150.000
300	A	3.2821	0.0834	150.000

4.3. Comparison of cost functions

This section compares results under the five different cost functions for energy efficiency investment. It focusses on how the mathematical form of the cost function, whether quadratic, exponential, or scaled by energy intensity, affects the firm behavior.

Figure 10: energy intensity under different cost functions

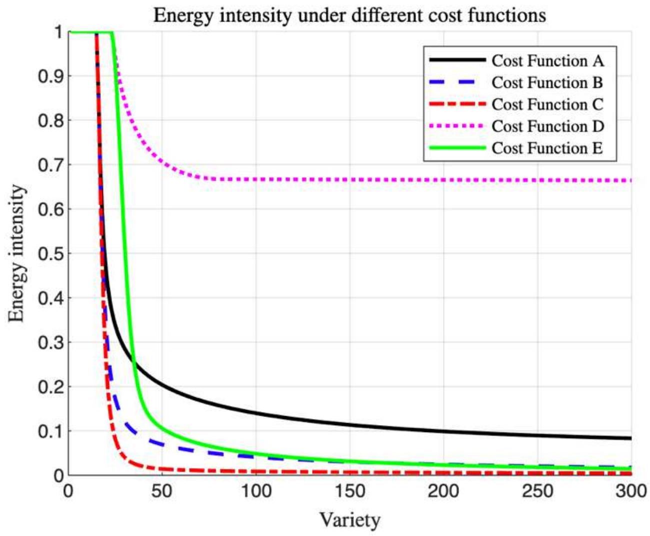


Figure 10 illustrates how energy intensity evolves across product varieties under five different cost functions. Initially, firms do not invest in efficiency, because energy remains cheap and the gains from reducing energy intensity are too small to justify the cost. As more varieties are introduced, pushing up the equilibrium energy price, investments in efficiency become more attractive. The extent and pattern of this decline, however, depend heavily on the cost structure faced by firms.

The figure shows that the strongest reductions in energy intensity occur under cost function C, followed closely by cost function E. Cost function B leads to a more moderate decline, while cost function A produces a smoother, gradual pattern that reflects more realistic diminishing returns to efficiency improvements. In contrast, cost function D yields the highest energy intensity across the board. For cost function D efficiency gains remain limited because the exponential cost structure makes deeper investments prohibitively expensive.

Importantly, none of the cost functions shown lead to the energy intensity of a product variety fully declining to zero. In all cases, energy intensity converges to a strictly positive value as the number of varieties increases. As emphasized by Bergmann and Kalkuhl (2025), this is crucial for long-run growth: under a binding energy constraint, sustained growth is only possible if energy intensity eventually approaches zero as the number of varieties grows. Otherwise, the economy will inevitably reach a limit on how many product varieties can be supported within the fixed energy budget.

A detail is that this asymptotic behavior, meaning the value energy intensity converges to, also depends on the parameter gamma in the cost function. For cost functions C and E, energy intensity can only converge to zero if gamma is exactly equal to one. In the current simulations, gamma is set to a value below one, which means that even under C and E, energy intensity declines strongly but remains positive. For the other cost functions, the structural form of the cost function still prevents energy intensity from reaching zero in the long run.

4.4. Policy scenario: investment subsidy

This part analyzes how an investment subsidy, which lowers $c(y_j)$, alters firm behavior. The goal is to evaluate whether a policy tool can support sustainable growth by accelerating energy efficiency improvements under a binding energy constraint.

Figure 11: energy intensity under cost function A vs A with subsidy

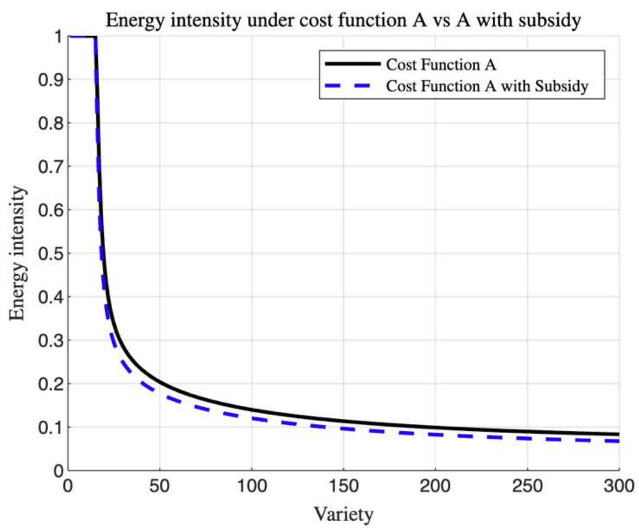


Figure 11 presents the evolution of energy intensity across product varieties for two scenarios: the baseline model with cost function A (black line), and a policy scenario in which a 30% investment subsidy is introduced (blue dashed line). In both cases, the model endogenously determines the investment level of the latest variety such that total energy use remains equal to the ceiling $\bar{E}$ =150.

As expected, energy intensity declines with each additional variety, since newly developed products are more energy-efficient by design. However, a key insight from figure 11 is that the subsidy consistently lowers energy intensity across the full range of varieties. The gap between the two curves shows that the subsidy accelerates the pace of technological improvement. This is especially visible between N = 30 and N = 100, where the difference between the two scenarios is most pronounced. By reducing the effective cost of investing in efficiency, the subsidy leads firms to allocate more resources toward lowering their energy intensity.

Table 3: outcomes with and without subsidy

Cost function A	Q	Energy intensity	Total profit	Total energy
Without subsidy (N=100)	0.8593	0.1399	480.95	150
With subsidy (N=100)	0.7624	0.1204	508.58	150

The comparison presented in Table 3 reveals several important insights into how the subsidy affects firm behavior and macroeconomic outcomes. Most notably, energy efficiency improves, the energy intensity of variety 100 decreases from 0.1399 in the baseline to 0.1204 in the subsidy scenario. This suggests that firms respond to the lower effective investment cost by choosing higher levels of efficiency. However, the improvement in energy intensity is much smaller than the 30% reduction in investment cost, reflecting diminishing returns to efficiency gains. As a result of these improved efficiency levels, the required energy price also declines from 0.8593 to 0.7624, which indicates that firms can produce at lower marginal costs while still complying with the energy ceiling.

Moreover, the subsidy leads to higher aggregate profit, which increases from 480.95 to 508.58, even though firms invest more. This reflects that the subsidy not only promotes environmental goals but also enhances private economic returns. Importantly, the total energy use remains unchanged at 150 in both scenarios. This confirms that the model's energy constraint is binding and endogenously enforced through the adjustment of the energy price in equilibrium. Together, these results illustrate that a well-designed investment subsidy can support both economic and environmental goals, improving efficiency and profitability without increasing energy use.

These results demonstrate that well-designed subsidies can improve both private and public outcomes. They increase firm-level profitability and enable higher output without exceeding environmental limits. This dynamic reflects the central idea in the green growth literature (e.g., Hepburn & Bowen, 2012) that policy can support economic expansion by driving technological progress and energy efficiency. The energy price in the model is not fixed, but endogenously determined in each scenario through a root-finding algorithm (*fsolve*) to satisfy the constraint:

$$E(t) = \int_0^{N(t)} \varepsilon_j * x_j(t)dj = \bar{E}$$

As the number of varieties increases, total energy demand rises, which requires a higher energy price to enforce the constraints. Which in turn encourages firms to reduce energy use per variety. This interaction illustrates how micro-level investment behavior is shaped by a macro-level environmental constraint.

Figure 12: energy intensity under cost function A vs A with subsidy, sigma = 1

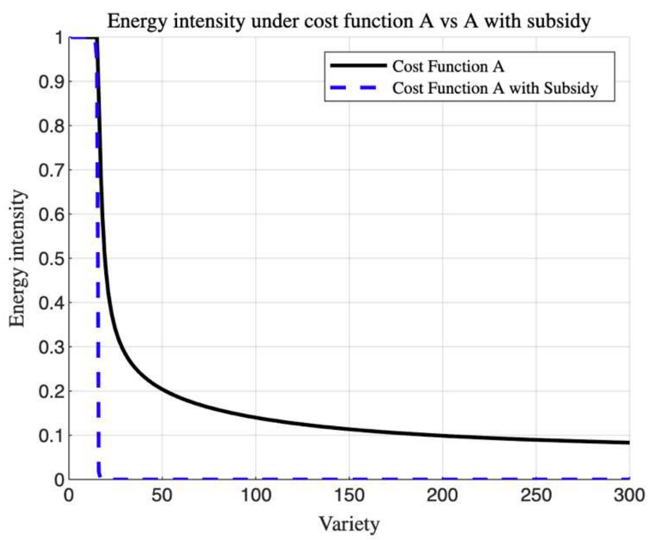


Figure 11 shows the effect of a full investment subsidy (σ = 1) on energy intensity across product varieties. With the subsidy, firms no longer face investment costs, which significantly changes their incentives. Energy efficiency continues to improve far beyond the baseline scenario, ultimately pushing energy intensity close to zero.

However, a key observation from the graph is that the effect of the subsidy only emerges gradually. In the early range of varieties, both scenarios show identical levels of energy intensity. This is because the earliest product varieties have relatively high energy intensity, and the marginal benefits of further investment are initially low, even when the marginal cost is zero. It is only after more varieties accumulate that the efficiency improvements compound and the subsidy-induced divergence becomes substantial.

This delayed effect reflects the dynamic nature of the model, where each new variety builds upon the efficiency level of the previous one. Since the first product types are still very energy inefficient, their high baseline energy intensity limits the effect of investment. But as energy intensity decreases, the marginal benefit of investing grows, and the subsidy begins to accelerate this process.

Although a full subsidy may appear highly effective in theory, it is not fiscally sustainable in practice. As energy intensity approaches zero, the required investment per variety increases exponentially. If every firm receives a full subsidy for these rising costs, total government spending would eventually exceed GDP. This illustrates a core trade-off, that while generous subsidies can push efficiency to the limit, their real-world application must be limited or conditional to remain both effective and financially viable.

4.5. Steady-state condition

As shown in Figure 10, firms initially do not invest in energy efficiency, energy intensity remains constant for the first few varieties. However, once the marginal benefits of investing in efficiency outweigh the costs, firms start to invest. This phase corresponds to the decreasing ε regime described in Proposition 4 of Bergmann & Kalkuhl (2025), in which the growth of the economy is sustained by increasingly energy-efficient products. During this phase, both product variety and output can still grow, even under a binding energy constraint $\bar{E}$, because each new variety requires less energy than the previous one.

Over time, however, the pace of improvement slows down. As the marginal cost of achieving further efficiency gains increases and the returns diminish, firms reduce their investment intensity. This results in a gradually flattening ε-curve, energy intensity continues to decline, but at a slower pace. This reflects the fundamental trade-off in firm decision-making between investment cost and the benefit of reduced energy use.

The curve approaches an asymptote, meaning that energy intensity converges toward a strictly positive value for this cost function. So, even though firms continue to invest marginally, ε does not go to zero, but stabilizes at a low, nonzero level. This outcome highlights that technological progress alone has limits when investment costs remain strictly positive.

Importantly, time is not explicitly modeled in this thesis. Instead, the simulation follows a sequence of product varieties until ε stabilizes. At this point, the model effectively enters a long-run regime in which further expansion of N would violate the energy ceiling unless ε declines further. Since this no longer happens, the economy reaches a steady state.

This corresponds to the steady-state solution from Proposition 2 in Bergmann & Kalkuhl (2025), in which energy intensity is constant and the energy constraint binds. In this regime, the growth rates of all key variables converge to zero:

$$g_N = g_c = g_Q = 0$$

The model equations in this steady state simplify to the functions below from Bergmann & Kalkuhl (2025):

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{\eta}[L\left(\frac{A\alpha^2}{\varepsilon Q(t)+1}\right)^{\frac{1}{1-\alpha}}\left(\frac{\varepsilon Q(t)+1-\alpha^2}{\alpha^2}\right) - \frac{c(t)}{N(t)}]$$

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}[\frac{(1-\alpha)\alpha AL}{\eta}\left(\frac{A\alpha^2}{\varepsilon Q(t)+1}\right)^{\frac{\alpha}{1-\alpha}} - \rho$$

The value of Q ensures that the right-hand side of these equations become exactly zero, thereby satisfying the binding energy constraint and locking the economy into a zero-growth path. The simulation thus illustrates that initial growth is driven by decreasing energy intensity through directed technical change. But once energy intensity stabilizes, at a low, but strictly positive level, the economy enters a steady state without further expansion.

5. Conclusion

This thesis explored whether long-run economic growth is feasible under a binding physical energy constraint and under what conditions firms voluntarily invest in energy efficiency. To answer this, the theoretical model of Bergmann & Kalkuhl (2025) was extended by endogenizing energy intensity. Firms now actively decide how much to invest in efficiency, based on energy prices and a convex cost function. The model thus integrates micro-level firm decisions into a macroeconomic framework with a physical energy ceiling and simulates long-run growth outcomes under various scenarios.

The main finding is that directed technical change, in the form of firm-level investment in energy efficiency, can sustain growth for a limited time under a physical energy constraint, but eventually reaches diminishing returns. In the free-market scenario where Q = 0, firms do not invest in energy efficiency, so energy intensity remains at 1. This implies that each variety consumes nearly one unit of energy, making sustained growth infeasible once the energy cap becomes binding.

When energy becomes scarce, the model endogenously determines the equilibrium price such that total energy demand equals the physical ceiling. This scarcity price enters firms' cost structures and incentivizes investment in efficiency. As a result, energy intensity declines across varieties. For many cost functions, including quadratic and exponential types, energy intensity drops from nearly 1 to values close to zero, allowing more firms to operate within the same energy limit.

These dynamics show that growth is limited not only by energy availability, but also by the feasibility of further efficiency gains. Once marginal costs become too high, investment halts, and growth flattens. The long-run potential of efficiency-driven growth thus depends critically on the cost function. Figure 10 illustrates this, cost functions C and E yield stronger improvements, but only under optimistic assumptions if $\gamma = 1$, energy intensity approaches zero, which is not realistic. A convex cost function with increasing, but not excessively steep, marginal costs is needed to balance feasibility and ambition.

The results also show that policy can extend the efficiency-driven phase. A modest subsidy of 30% lowers energy prices, increases investment, and raises output and profit without breaching the cap, though the effect remains limited. In contrast, a full subsidy (100%) drives energy intensity close to zero but results in extreme fiscal costs. This confirms the economic intuition that incentives must be carefully calibrated to avoid distortions.

These findings lead to three main theoretical contributions. First, they reinforce the directed technical change framework: firms respond to relative prices and policy signals, not abstract preferences for efficiency. This aligns with earlier work on how environmental and economic constraints shape innovation paths. Second, they show the importance of including physical limits in growth models, challenging assumptions of unlimited substitutability between inputs. Third, they demonstrate how endogenous firm behavior shapes macro-level sustainability outcomes and may produce very different long-run dynamics depending on cost structures and incentives.

In direct response to the central research question, this thesis finds that long-run growth under an energy constraint is only feasible during a transitional phase, as firms reduce energy intensity. Growth halts once further efficiency gains become too costly. Firms invest voluntarily only when energy is priced sufficiently high and cost structures permit affordable improvements. Policy tools like subsidies can support this transition but cannot avoid a plateau unless near-zero energy intensity is achieved, an unlikely outcome.

These insights are particularly relevant for climate policy and the energy transition. They underscore the importance of aligning private incentives with physical limits and of designing policies that reflect the technological diversity across firms. Sustainable growth in a resource-constrained world is possible, but only if economic behavior and policy design are grounded in realistic cost dynamics and biophysical boundaries.

6. Discussion

This thesis developed a theoretical model in which firms choose their energy intensity through investment, under a fixed energy ceiling. The simulations helped explore under what conditions firms invest in energy efficiency and whether such investment can support long-run growth. While the results offer relevant insights, the model also has important limitations that affect the scope and interpretation of the results.

First, the model is highly stylized. All firms are assumed to be identical, to have perfect information, and to invest without any friction. This is helpful for theoretical clarity, but it also makes the outcomes less realistic. In practice, firms differ in size, innovation capabilities, and access to finance and they respond differently to energy prices and policy incentives.

Second, the cost functions used to model investment behavior are not empirically grounded. The model uses mathematical functions, like quadratic or exponential to represent the cost of becoming more efficient. This help test different theoretical assumptions, but they are not based on real data. Some versions produce unrealistic outcomes, for example, energy intensities close to zero. That suggests the shape of the cost function plays a big role, and not all forms may be economically plausible.

Third, the model was not properly calibrated. Most parameters were chosen to illustrate mechanisms, not to match any real-world sector. While this is appropriate for a conceptual framework, it limits the empirical applicability of the results. Although some numerical outcomes may appear precise, they are best interpreted as illustrative rather than predictive.

Fourth, the analysis does not provide a full solution to the model, such as the transitional dynamics between steady states. The focus has been on long-run equilibrium outcomes under different policy and scarcity scenarios. While this offers valuable insights into steady state behavior, it leaves the question open of how the system adjusts over time, as exact transitional dynamics are not modeled.

To improve the model's realism and relevance, future research could go in several directions. One important step would be to calibrate the model using actual data from sectors. That would help test whether the simulated efficiency gains are achievable in practice. Another option would be to introduce firm heterogeneity in cost structures, innovation capacity, or access to capital. This would better reflect the diversity of real-world firms and help explain heterogenous responses to energy prices. Finally, incorporating uncertainty about future energy prices, investment outcomes, or behavioral frictions could yield more realistic dynamics and policy conclusions.

In short, this model provides a useful first step in understanding how firm behavior interacts with physical energy constraints and investment costs. While its simplifications limit its direct application to policy, it clarifies key mechanisms and lays the foundation for more empirically grounded models that can better support long-term decision-making.

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