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To Sign or Not to Sign: Social Contracts for Climate Mitigation and Intergenerational Transfers

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Abstract

Previous literature suggests that Pareto-improving climate mitigation is feasible if it is mixed with intergenerational transfers. We study the existence of Pareto-improving social contracts—consisting of climate mitigation efforts and intergenerational transfers—between two consecutive generations. We aim to analyze the Pareto-improving set of social contracts with different levels of intertemporal consumption substitutability. We also compare the Pareto-improving contracts' set under a capital income subsidy and a lump-sum transfer scheme. We employ an overlapping generations model in which households live for two periods: in the first period they work and earn labor income, and in the second period they retire and earn capital income from their savings. We find that, if discouraging saving is advantageous, a lower degree of intertemporal consumption substitutability increases the Pareto-improving set of social contracts. A savings reduction is beneficial if the capital accumulation externality is high and the interest rate is low compared to the population growth rate. Conversely, if a decrease in savings is undesirable, lowering the substitutability decreases the Pareto-improving contracts' set. We also find a similarity when comparing the two contract schemes. If a decrease in savings is advantageous, a scheme that results in a greater reduction in savings has a higher Pareto-improving contract's feasibility. Our findings therefore emphasize the importance of evaluating the effects of a savings reduction on the economy.

Keywords Overlapping generations models · Social contract · Climate mitigation · Intertemporal consumption substitutability · Pareto improvement · Capital income subsidy · Lump-sum transfer · Savings

Contents

1 Introduction

Climate change exposes our planet to greater risks of disaster, such as a rise in sea levels, wildfires, and the spread of diseases (Tol, 2019; Nordhaus, 2008). Climate change mitigation efforts are then necessary to reduce the climate damage risks in the future. While the efforts are performed currently, the fruits of such efforts will only become apparent after at least half a century (Nordhaus, 2013). As a result, climate policy is often seen as a trade-off between the current generation and future generations. This viewpoint may mislead policy decisions because it is possible to determine a policy mix—consisting of climate mitigations and intergenerational redistributions—in a Pareto-improving manner, that is, one's welfare could be better-off while holding the others' welfare constant (or even better-off) (e.g. Kotlikoff, Kubler, Polbin, Sachs, & Scheidegger, 2021; Dao, Burghaus, & Edenhofer, 2017; von Below, Dennig, & Jaakkola, 2016; Heijdra, Kooiman, & Ligthart, 2006; Bovenberg & Heijdra, 2002, 1998). Furthermore, utilizing intergenerational redistribution could increase the political feasibility of the climate mitigation efforts (Van der Meijden, Van der Ploeg, & Withagen, 2017).

We study the existence of Pareto-improving social contracts consisting of mitigation efforts and a pay-as-you-go (PAYG) pension system. At a certain period, the young generation transfers some amount to the old generation. The young generation then grows old, and they will also receive a transfer from the young generation *in that same period* (Heijdra, 2009). The transfer in a PAYG fashion therefore causes intergenerational redistributions. This scheme could then be combined with climate mitigation efforts to establish social contracts between generations.

This thesis follows Dao et al. (2017) who examine the existence of pareto-improving social contracts with intergenerational transfers based on a capital income subsidy scheme instead of lump-sum transfers. This would be interesting since previous literature mostly employed lump-sump intergenerational transfers in their models (e.g., Kotlikoff et al., 2021; von Below et al., 2016; Heijdra et al., 2006). We change the utility and the production function employed in Dao et al. (2017) into the constant elasticity of substitution (CES) function to generalize the model with differing elasticities of substitution. A different intertemporal consumption substitutability could affect the result regarding the existence of Pareto-improving social contracts. This is because intertemporal elasticity of substitution corresponds to a certain intertemporal inequality aversion (usually denoted by η), which is an important parameter in estimating optimal climate policy (Nordhaus, 2008). Furthermore, most of the empirically estimated intertemporal inequality aversion shows that individuals are more inequality averse than logarithmic utility ($\eta = 1$). For example, Drupp, Freeman, Groom, and Nesje (2018) estimate the mean of $\eta = 1.35$.

A meta-analysis by Groom and Maddison (2013) results in $\eta = 1.5$ as their best-guess estimate.

We also develop an overlapping generations (OLG) model for a lump-sum transfer scheme, which is a modification of the OLG model for a capital income subsidy scheme employed by Dao et al. (2017). This allows us to compare these two schemes. We are particularly interested in whether one of the schemes has a higher of Pareto-improving set of social contracts. A larget set of Pareto-improving contracts improves the feasibility for a social contract to be established.

Two research questions are formulated. (1) What are the effects of intertemporal consumption substitutability on the existence of Pareto-improving social contracts? (2) How does a different scheme of intergenerational transfers, capital income subsidy and lump-sum, affect the feasibility of Pareto-improving social contracts? We therefore have two main findings for this thesis. First, lowering intertemporal consumption substitutability increases the feasibility of the existence of Pareto-improving social contracts provided that discouraging savings is beneficial. Second, a scheme which causes a stronger decrease in savings has a higher Pareto-improving contracts feasibility if a decrease in savings is beneficial.

In the next section, we will review past studies that have used OLG models to analyze the climate change or stock pollution case. Section 3 provides the model used for this thesis, which mainly follows Dao et al. (2017) except that we use CES utility and production function. We also develop the model under the lump-sum transfer scheme. Section 4 discusses the calibrations and results of the models. We conclude this thesis in section 5 and describe its limitations.

2 Literature Review

The model employed for this thesis is called an overlapping generations (OLG) model. The OLG model assumes that generations are selfish, so they act to maximize their own lifetime welfare without taking into account the welfare of future generations. In contrast, infinitely-lived agent (ILA) models assume altruistic agents, which explains bequests from the old generations to the young generations (Heijdra, 2009). The OLG model is suitable for analyzing climate policy because it involves multiple generations, whose decisions may not consider the well-being of their future descendants (Gerlagh & van der Zwaan, 2001).

One of the workhorses of the OLG model was formulated by Samuelson (1958). He developed a discrete-time OLG model that describes the consumption and savings decisions during the lifetime of a representative agent. He concluded that under an OLG model, there exists dynamic inefficiency because the young generation over-save. The over-saving could be reduced by implementing a PAYG social security system because the PAYG pension transfers will be received, and these transfers would act as a substitute for the young agent's savings. Because the PAYG system implies intergenerational transfers, this instrument could be used to redistribute the welfare effects of climate policy. Therefore, this OLG model with PAYG transfer schemes is often analyzed in the past literature concerning climate change (e.g., Dao et al., 2017; von Below et al., 2016).

The existing literature studying intergenerational distribution and climate change varies in their modelling assumptions. Foley (2009) assumes that the economy is dynamically inefficient because there is an intergenerational externality due to climate change. In this case, there are misallocations in the investment decision, that is, the present generation overinvests in physical capital. He then suggests the combination of climate mitigation and public debt, while noting that raising public debt will increase the interest rate and crowd-out conventional investment. Bovenberg and Heijdra (2002, 1998) draw the same conclusion regarding a pollution stock problem using an OLG model. They also assume an endogenous interest rates and propose that public debt policy with a capital income tax could accomplish a Pareto-improvement between generations. Heijdra et al. (2006) utilized an OLG model in the context of a small-open economy where interest rates are exogenous, and their model yields similar results that the policy mix of capital income tax and public debt leads to pareto-improving welfare. Kotlikoff et al. (2021) adopted an OLG model and combined it with the DICE model from Nordhaus (2013) to provide a more reliable translation of climate change damage to the economy. They also agree with the discussed literature above, and they also propose a policy mix of carbon tax and public debt which could result in uniform welfare gains for all generations.

Other than through public debt, welfare between generations could also be redistributed via the PAYG green pension scheme. Such a scheme was studied using an OLG model by Dao et al. (2017) and von Below et al. (2016). Both of them involve intergenerational contracts between generations at successive periods. The agreement consists of an emission limit and lump-sum pension transfers in von Below et al. (2016), while Dao et al. (2017) consider mitigation efforts and a capital income subsidy. Nevertheless, both studies show the same result: they prove the existence of pareto-improving intergenerational contracts consisting of a combination of climate mitigation efforts and pension transfers. However, the existence of Pareto-improving social contracts in Dao et al. (2017) requires that the net income be high enough, surpassing a certain net income threshold. Their result stems from their model specification that climate mitigation is carried out as a share of net income.

All the previous literature reaches the same consensus regarding the potential of utilizing either public debt or a green pension scheme to achieve a Pareto-improving climate policy, with some exceptions in Dao et al. (2017). Therefore, we employ the OLG model based on Dao et al. (2017) but with a more general utility and production function. Instead of using a logarithmic utility function as in Dao et al. (2017), we specify a CES utility function. This allows us to examine the effects of intertemporal elasticity of substitution on the existence of Pareto-improving social contracts. Likewise, we also conduct a more generalized version of the production function by using a CES production function rather than a Cobb-Douglas function. We therefore contribute to the literature pertaining to intergenerational distribution and climate change with pension transfers, which is relatively more scarce compared to public debt instruments.

3 The Model

3.1 Capital Income Subsidy Scheme

The OLG model for the intergenerational social contracts with capital income subsidy scheme is shown in Figure 1. Because an agent lives in two periods, there are two generations that overlap in the same period. In Figure 1, *old generation* $\mathcal{G}_0$ coexists with *young generation* $\mathcal{G}_1$ in period 1. An intergenerational social contract consists of two commitments: an investment for mitigation denoted by the mitigation share m_t and a transfer rate for the old generation as a capital income subsidy τ^o_{t+1} . If the contract (m_0, τ^o_1) is signed between $\mathcal{G}_0$ and $\mathcal{G}_1$, the mitigation share m_0 must be performed by $\mathcal{G}_0$ by spending a share of their labor income w_0. The total cost for $\mathcal{G}_0$ is $m_0 w_0$, while the benefit is the total amount of capital income subsidy $R_1 k_1 \tau^o_1$. The terms R_t and k_t represent the return to capital and per capita investment, respectively. Because we assume zero population growth and the transfers are formulated in PAYG fashion, the total amount paid by $\mathcal{G}_1$ and received by $\mathcal{G}_0$ must be equal ($\tau^y_1 w_1 = R_1 k_1 \tau^o_1$).[1] The benefit for $\mathcal{G}_1$ is a higher labor income due to the climate mitigation performed in period 0. In the beginning of period 1, if $\mathcal{G}_1$ and $\mathcal{G}_2$ also signed the contract (m_1, τ^o_2), the transfer and the mitigation commitment are also undertaken in the same manner. The climate mitigation must be performed by $\mathcal{G}_1$ in period 1, and they will receive the capital income subsidy in period 2. In period 2, the succeeding generation $\mathcal{G}_2$ has to pay for the capital income subsidy.

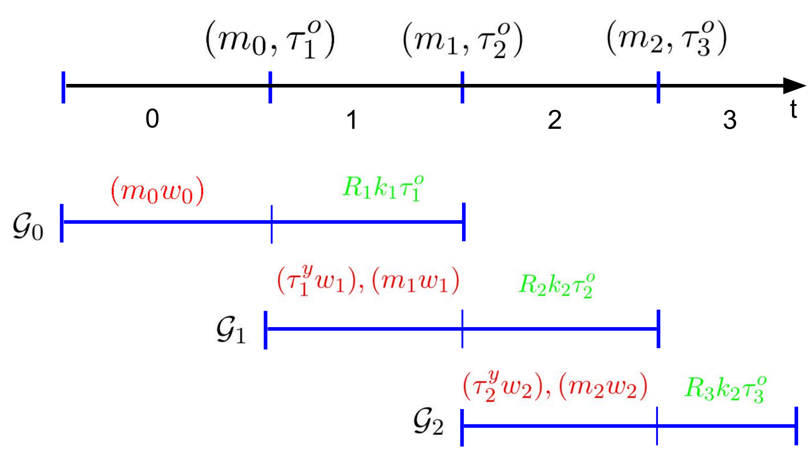


Figure 1: Intergenerational social contracts for capital subsidy scheme

[1] If we assume nonconstant population, the transfers equality become $L_0(1+n)\tau^y_1 w_1 = L_0 R_1 k_1 \tau^o_1$, where n is the population growth rate.

3.1.1 The Household

A representative agent of $\mathcal{G}_t$ has the following CES utility function:

$$\mathcal{V}_t = \frac{(c_t^y)^\theta}{\theta} + \beta\frac{(c_{t+1}^o)^\theta}{\theta}; \text{ with } \theta \leq 1 \text{ and } \theta \neq 0, \tag{3.1}$$

where c_t^y is the consumption of a representative agent when they are young at period t. c_{t+1}^o is their consumption when they are old at period $t+1$. The parameter β represents the utility discount factor for the individual. The parameter θ denotes substitution parameter, and it corresponds to the *intertemporal elasticity of substitution* given by $\sigma_u = 1/(1-\theta)$. A higher θ means that substituting consumption between the two periods becomes easier. The second interpretation of θ is that θ also reflects an *intertemporal inequality aversion* calculated by $\eta = 1 - \theta$ (Nordhaus, 2008, p. 173). Households that could easily substitute their intertemporal consumption implies that they have a low intertemporal inequality aversion. We also assume perfectly inelastic labor supply.

The representative household faces a budget constraint on their lifetime consumption. When they are young, the amount of consumption c_t^y and savings k_{t+1} must be equal to their wages after paying for transfer rate τ_t^y and mitigation share m_t. When the agent becomes old, at period $t+1$, they will receive the capital income subsidy at the rate τ_{t+1}^o on top of the return on capital R_{t+1} for their savings k_{t+1}. Therefore, we can write the budget constraints:

$$c_t^y + k_{t+1} = I_t(1 - m_t), \tag{3.2}$$

and

$$c_{t+1}^o = R_{t+1}^e k_{t+1}(1 + \tau_{t+1}^o), \tag{3.3}$$

where I_t represents net income after the transfer to the old agents at period t, that is, $I_t = w_t(1-\tau_t^y)$. The right-hand side of equation (3.2) is *disposable income* $I_t(1-m_t)$: net income less mitigation expenses. There are two assumptions for the formulation of the budget identities. First, we assume that capital stock fully depreciates in each period, and therefore the capital stock K_{t+1} comes directly from total savings in period t. This implies $S_t L = K_{t+1}$, so we could then interpret k_t both as savings and capital per capita K_t/L. Second, we assume that the agent has perfect foresight, which results in $R_{t+1}^e = R_{t+1}$ where the expected interest rate converges with the interest rate at period $t+1$.

The agent maximizes (3.1) subject to (3.2) and (3.3), and this yields the optimal consumption and savings decisions:[2]

$$k_{t+1} = s(R_{t+1}, \tau_{t+1}^o) I_t(1 - m_t), \tag{3.4}$$

[2]See appendix A.1 for the derivation details.

$$c_t^y = \left[1 - s(R_{t+1}, \tau_{t+1}^o)\right] I_t(1 - m_t), \tag{3.5}$$

and

$$c_{t+1}^o = s(R_{t+1}, \tau_{t+1}^o) I_t(1 - m_t) R_{t+1}(1 + \tau_{t+1}^o), \tag{3.6}$$

where $s(R_{t+1}, \tau_{t+1}^o)$ denotes the optimal savings rate of the agent:

$$s(R_{t+1}, \tau_{t+1}^o) = \left(\frac{1}{1 + [\beta^{\frac{1}{\theta}} R_{t+1}(1 + \tau_{t+1}^o)]^{\frac{\theta}{\theta-1}}}\right). \tag{3.7}$$

This savings rate equation is the main difference between our household model and that of Dao et al. (2017). Their assumption of logarithmic utility yields a constant savings rate, whereas the savings rate in our model is affected by the interest rate R_{t+1} and transfer rate τ_{t+1}^o.[3] The sign of the effects depends on the parameter θ. If $0 \leq \theta < 1$, it yields positive effects on the savings rate, while negative effects occur if $\theta < 0$.[4]

3.1.2 The Firm and the Pollution Stock

For the models of pollution stock and pollution flows, we follow the formulations from Dao et al. (2017):

$$P_t = \xi K_t - \gamma M_t; \;\; \xi > 0; \; \gamma > 0. \tag{3.8}$$

The pollution flow at period t, P_t, is positively proportional to the capital stock K_t and negatively proportional to the mitigation effort M_t. The parameters ξ and γ represent coefficients which translate the capital stock and mitigation effort to the pollution flow, respectively. The mitigation effort is equal to the mitigation share in the prior period $t-1$ times net income $M_t = m_{t-1} I_{t-1}$. This specification reflects that the mitigation benefits are delayed by one period. For the pollution stock, we write the following function:

$$E_t = (1 - \delta) E_{t-1} + \xi K_t - \gamma M_t; \text{ with } 0 \geq \delta \geq 1, \tag{3.9}$$

where δ denotes the decay rate parameter of the pollution stock in the atmosphere. This pollution stock function stems from the assumption that pollution only comes from the capital stock. This linear specification of pollution stock is also similar to studies by

[3] Logarithmic utility in Dao et al. (2017) has savings rate $s = \frac{\beta}{1+\beta}$. Our model yields the same result if we impose $\theta = 0$.

[4] The partial derivative for savings rate with respect to interest rate is as follows:

$$\frac{\partial s(R_{t+1}, \tau_{t+1}^o)}{\partial R_{t+1}} = \frac{\theta}{\theta - 1} \frac{[s(R_{t+1}, \tau_{t+1}^o)^2 - s(R_{t+1}, \tau_{t+1}^o)]}{R_{t+1}}.$$

The effect of τ_{t+1}^o on $s(R_{t+1}, \tau_{t+1}^o)$ could be derived in the same way, and it has the same sign.

Dao and Davila (2014) and Tabellini (1991). The difference with our model is that the former assumes pollution originates from both consumption and production, while the latter assumes pollution comes from consumption. Next, we specify a CES production function with a constant return to scale:

$$Y_t = z(E_{t-1})[\alpha K_t^\rho + (1-\alpha)L_t^\rho]^{\frac{1}{\rho}};\ \ 0 \geq \alpha \geq 1;\ \rho \leq 1 \text{ and } \rho \neq 0, \tag{3.10}$$

where $z(E_{t-1})$ is the total factor productivity in period t, which is negatively affected by the pollution stock E_{t-1}. The parameter α reflects the weight of capital and labor. The specification of the total factor productivity is as follows:

$$z(E_t) = Ae^{-|E_t|};\ \ A > 0. \tag{3.11}$$

This functional form implies that climate damage raises at an exponential rate when the greenhouse gas concentrations in the atmosphere increase. Given the production function above, we could derive the profit maximizing decisions regarding the capital and labor. The resource demands are chosen such that their costs (interest rate and wage rate) are equal to their respective marginal productivities:[5]

$$R_t = \alpha z(E_{t-1}) \left[\alpha + (1-\alpha)k_t^{-\rho}\right]^{\frac{1-\rho}{\rho}}, \tag{3.12}$$

and

$$w_t = (1-\alpha)z(E_{t-1})\left[\alpha k_t^\rho + 1 - \alpha\right]^{\frac{1-\rho}{\rho}}, \tag{3.13}$$

where R_t is the rate of return on capital, and w_t is the wage rate at period t. The parameter ρ is analogous to θ in the utility function, which reflects elasticity of substitution between capital and labor $\sigma_p = 1/(1-\rho)$. Both (3.12) and (3.13) are written in intensive form, with $k_t = K_t/L$. It is also useful to determine the ratio of R_t and w_t to eliminate most of the term complexities in the model:

$$k_t^{\rho-1} = \frac{1-\alpha}{\alpha}\frac{R_t}{w_t}. \tag{3.14}$$

3.1.3 Equilibrium

In the market equilibrium, we have a system of equations which incorporates the optimal behaviour in the household and the firm, with the additional pollution function and the government's budget balance condition. Equations (3.4) through (3.7) represent the optimal consumption and savings decisions of the households. In equilibrium, the endogenous

[5] See Appendix A.2 for the derivation details.

interest rate R_{t+1} must be equal to the marginal product of capital specified in equation (3.12), by adding the subscripts by one period.

We summarise the system of equations representing market equilibrium in Table 1.

Table 1: System of Equations for Capital Income Subsidy Scheme

$$c_t^y = \Big(1 - s(R_{t+1}, \tau_{t+1}^o)\Big) I_t (1 - m_t) \tag{3.15}$$

$$c_{t+1}^o = s(R_{t+1}, \tau_{t+1}^o) I_t (1 - m_t) R_{t+1} (1 + \tau_{t+1}^o) \tag{3.16}$$

$$k_{t+1} = s(R_{t+1}, \tau_{t+1}^o) I_t (1 - m_t) \tag{3.17}$$

$$s(R_{t+1}, \tau_{t+1}^o) = \left(\frac{1}{1 + [\beta^{\frac{1}{\theta}} R_{t+1} (1 + \tau_{t+1}^o)]^{\frac{\theta}{\theta - 1}}} \right) \tag{3.18}$$

$$I_t = (1 - \alpha) z(E_{t-1}) \left[\alpha k_t^\rho + (1 - \alpha)\right]^{\frac{1-\rho}{\rho}} \left(1 - \frac{\alpha}{1 - \alpha} k_t^\rho \tau_t^o \right) \tag{3.19}$$

$$R_{t+1} = \alpha z(E_t) \left[\alpha + (1 - \alpha) k_{t+1}^{-\rho}\right]^{\frac{1-\rho}{\rho}} \tag{3.20}$$

$$E_t = (1 - \delta) E_{t-1} + \xi k_t - \gamma m_{t-1} I_{t-1} \tag{3.21}$$

$$z(E_t) = A e^{-|E_t|} \tag{3.22}$$

These general equilibrium equations have nonlinear simultaneous equations of (3.17) and (3.20), and thus we cannot solve it analytically for k_{t+1} and R_{t+1}. The government's budget balance condition must hold. We substitute the ratio of R_t and w_t from (3.14) to obtain the budget balance condition:

$$\tau_t^y = \frac{\alpha}{1 - \alpha} k_t^\rho (\tau_t^o). \tag{3.23}$$

Noting that $I_t = w_t(1 - \tau_t^y)$ and that wages w_t must satisfy equation (3.13) in equilibrium, we then could derive the net income function (3.19). We also need to incorporate the pollution stock function and the total factor productivity, which we rewrite in equations (3.21) and (3.22).

We conduct a comparative-static analysis to study the effects of social contracts (m_t, τ_{t+1}^o) on the households' consumption-savings decisions. We provide the details of deriving the partial derivatives in Appendix A.3.

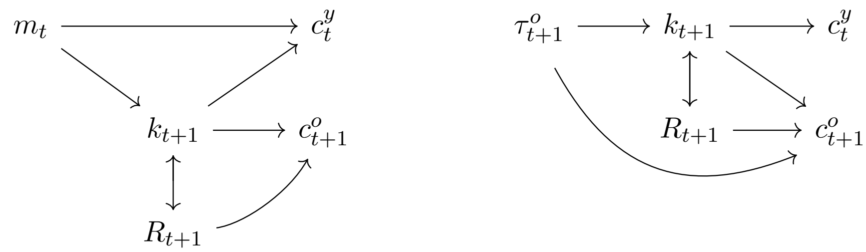


Figure 2: Effects of Social Contracts with Capital Income Subsidy Scheme

Figure 2 illustrates different channels regarding the effects of the capital income subsidy social contracts for generation t. Mitigation share m_t affects the young-age consumption c_t^y directly via the reduction of disposable income $I_t(1-m_t)$ and indirectly through changes in savings k_{t+1}. The old-age consumption is affected by the mitigation share m_t through the changes in savings k_{t+1} and R_{t+1}. The transfer rate τ_{t+1}^o only affects the young-age consumption c_t^y indirectly via savings k_{t+1}, but τ_{t+1}^o also affects old-age consumption c_{t+1}^o directly. The double arrow between R_{t+1} and k_{t+1} reflects their simultaneity effects on each other.

First, we consider the effects of mitigation share m_t on the optimal savings k_{t+1}:

$$\frac{\partial k_{t+1}}{\partial m_t} = -\frac{s(R_{t+1}, \tau_{t+1}^o) I_t}{1 + \frac{\theta}{\theta-1}\varepsilon_{R,k}\left(1 - s(R_{t+1}, \tau_{t+1}^o)\right)} < 0, \tag{3.24}$$

where $\varepsilon_{R,k}$ is the elasticity of R_{t+1} with respect to k_{t+1}.[6] The sign of the partial derivative (3.24) is always negative because its denominator is always positive. If $0 < \theta < 1$, the denominator in (3.24) must be positive because $\varepsilon_{R,k} < 0$ and $1 - s(R_{t+1}, \tau_{t+1}^o) > 0$, and thus the effect of m_t on k_{t+1} is negative. The negative value of $\varepsilon_{R,k}$ is due to diminishing marginal return on capital. Imposing $\theta = 0$ also yields a negative sign of (3.24) with a proportional effect of $-sI_t$. If we impose $\theta < 0$, the denominator becomes smaller, and it lowers the partial derivative.[7]

For the intuitive interpretation of (3.24), we emphasize again that the parameter θ reflects the substitutability of consumption. Suppose $0 < \theta < 1$, it means that the substitution between the current and future consumption is relatively easier compared to Cobb-Douglas utility case. Due to an increase in the mitigation share m_t, savings k_{t+1} decrease, but this

[6] The elasticity of capital per capita with respect to the interest rate is derived as the following expression:

$$\varepsilon_{R,k} = \frac{(\rho-1)(1-\alpha)}{\alpha k_{t+1}^{\rho} + 1 - \alpha} < 0; \text{ with } \rho \leq 1$$

[7] We can rule out a negative denominator because the term $\frac{\theta}{\theta-1}\varepsilon_{R,k}\left(1 - s(R_{t+1}, \tau_{t+1}^o)\right)$ cannot be lower than -1. This is because a higher $\varepsilon_{R,k}$ also implies a higher $s(R_{t+1}, \tau_{t+1}^o)$, so $|\varepsilon_{R,k}\left(1 - s(R_{t+1}, \tau_{t+1}^o)\right)| < 1$.

decrease in savings also leads to an increase in the interest rate R_{t+1}. The households then increase their savings rate, which offsets the first effect of the decrease in savings k_{t+1}. In a relatively difficult substitution ($\theta < 0$), an increase in interest rate decreases savings rate. Households are reluctant to sacrifice the current consumption with the future consumption, so they compensate their lower income with lower savings rate. This reduction in savings rate thus lowers savings k_{t+1} even further. Hence, this implies that a lower θ yields a lower partial derivative (3.24).

Next, we derive the effects of mitigation share m_t on the optimal consumption:

$$\frac{\partial c_t^y}{\partial m_t} = -I_t - \frac{\partial k_{t+1}}{\partial m_t} \lesseqgtr 0, \tag{3.25}$$

and

$$\frac{\partial c_{t+1}^o}{\partial m_t} = \frac{\partial k_{t+1}}{\partial m_t}(\varepsilon_{R,k} + 1)R_{t+1}(1 + \tau_{t+1}^o) \lesseqgtr 0. \tag{3.26}$$

For the effect of mitigation share m_t on consumption when the agent is young, c_t^y, the sign of the partial derivative (3.25) is negative if the decrease in net income I_t surpass the decrease in savings. This negative effect occurs for households who are willing to reduce their young-age consumption (that is, when θ is high). The households who are very reluctant to reduce their young-age consumption would greatly reduce their savings. This greater savings reduction, combined with a high $|\varepsilon_{R,k}|$, could reverse the sign of expression (3.25) from negative to positive. The importance of $|\varepsilon_{R,k}|$ is that it affects households' lifetime budget. A high $|\varepsilon_{R,k}|$ would increase interest rate R_{t+1} greatly if the households reduce savings, and therefore increase the budget for their old-age consumption.

The sign of the partial derivative (3.26) is also negative if we assume the elasticity $|\varepsilon_{R,k}| < 1$. A high elasticity $|\varepsilon_{R,k}| > 1$ could occur if the factor substitutability ρ is very low and when k_{t+1} is high.[8] Because the mitigation share m_t affects the old-age consumption c_{t+1}^o through savings k_{t+1} and interest rate R_{t+1}, the size of the reduction in c_{t+1}^o largely depends on the parameter θ. A lower θ implies a higher reduction in savings k_{t+1}, and then this causes a greater reduction in the old-age consumption. The effect of mitigation share m_t on the old-age consumption is positive when $|\varepsilon_{R,k}| > 1$ because of even if they have lower savings, they still receive higher total return on their savings k_{t+1}.

For the effect of the transfer rate τ_{t+1}^o on the optimal consumption-savings decisions, we

[8]For Cobb-Douglas production function, $\varepsilon_{R,k} = \alpha - 1 > -1$. Therefore, $|\varepsilon_{R,k}| < 1$ is reasonable if the factor substitutability ρ is around 0. If ρ is negative, the savings k_{t+1} must also be high enough to yield $\varepsilon_{R,k} < -1$.

derive the following partial derivatives:

$$\frac{\partial k_{t+1}}{\partial \tau^o_{t+1}} = -\frac{\theta}{\theta-1} k_{t+1} \left(\frac{1-s(R_{t+1},\tau^o_{t+1})}{(1+\tau^o_{t+1})} \right) \lesseqgtr 0, \tag{3.27}$$

$$\frac{\partial c^y_t}{\partial \tau^o_{t+1}} = -\frac{\partial k_{t+1}}{\partial \tau^o_{t+1}} \lesseqgtr 0, \tag{3.28}$$

and

$$\frac{\partial c^o_{t+1}}{\partial \tau^o_{t+1}} = R_{t+1}k_{t+1} \left(1 - \frac{\theta}{\theta-1} \left(1-s(R_{t+1},\tau^o_{t+1})\right)\left(\varepsilon_{R,k}+1\right) \right) > 0. \tag{3.29}$$

The signs of the partial derivatives (3.27) and (3.28) hinge on the parameter θ. Households with a relatively flexible intertemporal consumption, with $0 < \theta < 1$, will increase their savings k_{t+1} to earn a higher old-age consumption. The transfer rate τ^o_{t+1} does not affect savings k_{t+1} if $\theta = 0$. Households with inflexible consumption substitution (when $\theta < 0$) would instead reduce their savings to increase their young-age consumption and balance with the foreseen increase in their old-age consumption.

The effects of an increase in the transfer rate τ^o_{t+1} have an identical nature to an increase in real interest rate R_{t+1}. The effects of an increase in the interest rate R_{t+1} could be divided into substitution effect and income effect (Romer, 2018, p. 80). The substitution effect encourages the households to reduce their young-age consumption because its relative cost raises compared to old-age consumption. As a result, the substitution effect reduces their young-age consumption while their old-age consumption increases. The income effect increases both young-age and old-age consumption because the households' lifetime budget increases.[9] These two effects work in opposite directions for the young-age consumption, and the total effect depends on the value of θ. The substitution effect dominates if $0 < \theta < 1$ because this household would save more to increase the relatively cheaper old-age consumption. On the contrary, the income effect is stronger than the substitution effect for households who are less willing to substitute (that is, when $\theta < 0$). The subtitution effect and the income effect work in the same direction for the household's old age consumption; hence, the sign of the partial derivative (3.29) is unambiguous.

[9] This effect applies only to households who save, which is implicitly assumed in our model with non-negative k_{t+1}. For borrowers, the income effect decreases both their young-age and old-age consumption.

3.2 Lump-sum Transfer Scheme

The OLG model for intergenerational social contracts with a lump-sum transfer scheme is similar to the capital income subsidy scheme. Let there be two successive generations, generation t ($\mathcal{G}_t$) and generation $t+1$ ($\mathcal{G}_{t+1}$), signing a social contract with a lump-sum transfer scheme $(m_t, \mathcal{T}_{t+1})$. By committing to this contract, households $\mathcal{G}_t$ agree to carry out mitigation share m_t financed from their labor income when they are young. In return, households $\mathcal{G}_t$ will receive a lump-sum transfer $\mathcal{T}_{t+1}$ when they are old. The lump-sum transfer is deducted from $\mathcal{G}_{t+1}$'s labor income. Because we assume zero population growth, the amount of transfers paid by *each* household $\mathcal{G}_{t+1}$ is the same as the amount received by *each* household $\mathcal{G}_t$. A positive population growth would reduce the payment paid by each household $\mathcal{G}_{t+1}$ because they have a larger population. We illustrate this contract in Figure 3.

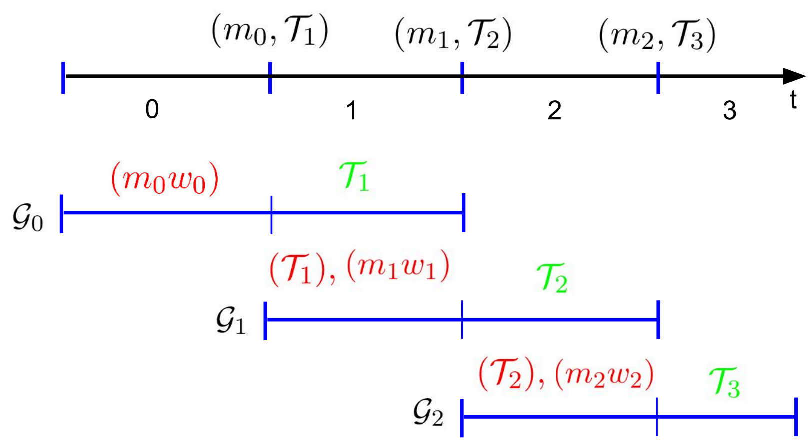


Figure 3: Intergenerational social contracts for lump-sum transfer scheme

3.2.1 The Household

The utility function for the representative household is the same as the utility function (3.1) under the capital income subsidy scheme. For the budget constraints, we change the expressions (3.2) and (3.3) into the budget constraints for a lump-sum transfer scheme:

$$c_t^y + k_{t+1} = w_t(1 - m_t) - \mathcal{T}_t, \tag{3.30}$$

and

$$c_{t+1}^o = R_{t+1}k_{t+1} + \mathcal{T}_{t+1}. \tag{3.31}$$

The only difference here is that the transfer is not accounted as a percentage of neither labor income w_t nor capital income $R_{t+1}k_{t+1}$. Instead, the transfer $\mathcal{T}_t$ to the coexisting old generation is deducted from the young households' disposable income at period t. In period $t+1$, the households receive lump-sum transfer $\mathcal{T}_{t+1}$. We still make the same assumption that the mitigation effort is determined as a share of labor income $w_t m_t$. All other assumptions from budget identities (3.2) and (3.3) are also applicable to the lump-sum lifetime budget constraint. We assume that the household is blessed with perfect foresight, the capital stock is fully-depreciated in each period, and the population is constant. We therefore have the household's utility maximization problem of (3.1) subject to (3.30) and (3.31).

3.2.2 The Firm, the Pollution Stock, and Equilibrium

For the firm's profit maximization problem, we have the same result as in equations (3.12) and (3.13) for the interest rate and labor income, respectively. The pollution stock and total factor productivity specifications also follow expressions (3.9) and (3.11), respectively. Table 2 shows the equilibrium conditions under the social contract with a lump-sum transfer scheme.

Table 2: System of Equations for Lump Sum Transfers

$$k^*_{t+1} = \frac{w_t(1-m_t) - \mathcal{T}_t - \mathcal{T}_{t+1}(\beta R_{t+1})^{\frac{1}{\theta-1}}}{1 + (\beta^{\frac{1}{\theta}} R_{t+1})^{\frac{\theta}{\theta-1}}} \tag{3.32}$$

$$c^y_t = w_t(1-m_t) - \mathcal{T}_t - k^*_{t+1} \tag{3.33}$$

$$c^o_{t+1} = R_{t+1} k^*_{t+1} + \mathcal{T}_{t+1} \tag{3.34}$$

$$w_t = (1-\alpha) z(E_{t-1}) \left[\alpha (k_t)^{\rho} + (1-\alpha)\right]^{\frac{1-\rho}{\rho}} \tag{3.35}$$

$$R_{t+1} = \alpha z(E_t) \left[\alpha + (1-\alpha)(k^*_{t+1})^{-\rho}\right]^{\frac{1-\rho}{\rho}} \tag{3.36}$$

$$E_t = (1-\delta)E_{t-1} + \xi k_t - \gamma w_{t-1} m_{t-1} \tag{3.37}$$

$$z(E_t) = A e^{-|E_t|} \tag{3.38}$$

The household's utility maximization problem yields optimal consumption-savings decisions (3.32), (3.33), and (3.34).[10] We denote the equilibrium savings as k^*_{t+1} to differentiate it from the nonequilibrium savings k_{t+1}.

[10] See Appendix B.1 for the derivations.

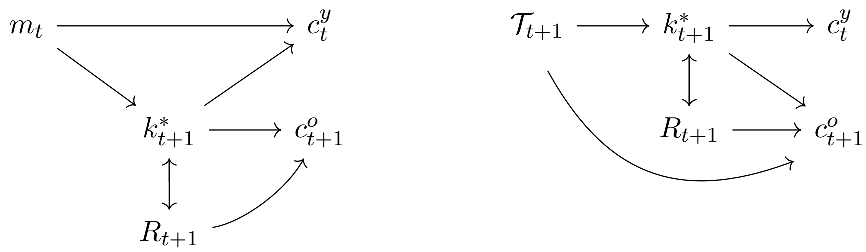


Figure 4: Effects of Social Contracts with Lump-sum Transfer Scheme

Comparative-static analysis is also required to analyze the effects of the lump-sum social contract $(m_t, \mathcal{T}_{t+1})$ on the household's consumption-savings decisions. We show the different mechanisms of the effects of the social contract in Figure 4. This figure is identical with Figure 2 except with a few changes in the notations; therefore, the mechanisms on how the social contracts affect consumption-savings is identical for both lump-sum transfer scheme and capital income subsidy scheme. We also derive the partial derivatives for a lump-sum transfer scheme. Appendix B.2 explains the derivations of the partial derivatives 3.39 to 3.44.

For the effects of the mitigation share m_t for the lump-sum transfer scheme, we have the following partial derivatives:

$$\frac{\partial k_{t+1}^*}{\partial m_t} = -\frac{w_t}{1 + (\beta^{\frac{1}{\theta}} R_{t+1})^{\frac{\theta}{\theta-1}} \left[1 + \frac{\theta}{\theta-1}\varepsilon_{R,k^*}\left(1 + \frac{1}{\theta}\frac{\mathcal{T}_{t+1}}{k_{t+1}^* R_{t+1}}\right)\right]} < 0, \tag{3.39}$$

$$\frac{\partial c_t^y}{\partial m_t} = -w_t - \frac{\partial k_{t+1}^*}{\partial m_t} \lesseqgtr 0, \tag{3.40}$$

and

$$\frac{\partial c_{t+1}^o}{\partial m_t} = \frac{\partial k_{t+1}^*}{\partial m_t} R_{t+1}\left(\varepsilon_{R,k^*} + 1\right) \lesseqgtr 0. \tag{3.41}$$

These partial derivatives have similar signs with the partial derivatives for capital income subsidy scheme. Note that if we impose the same parameters and zero transfers, the partial derivatives (3.27), (3.28), and (3.29) coincide with (3.39), (3.40), and (3.41), respectively.[11] Because the denominator of the expression (3.39) is always positive, the mitigation share's effect on savings k_{t+1} is negative.

[11] Suppose we impose $\tau_t^y = \mathcal{T}_t = \tau_{t+1}^o = \mathcal{T}_{t+1} = 0$. Then net income would be $I_t = w_t$. Both partial derivatives (3.24) and (3.39) reduce to the following expression:

$$\frac{\partial k_{t+1}}{\partial m_t} = -\frac{w_t}{1 + (\beta^{\frac{1}{\theta}} R_{t+1})^{\frac{\theta}{\theta-1}} + \frac{\theta}{\theta-1}\varepsilon_{R,k}(\beta^{\frac{1}{\theta}} R_{t+1})^{\frac{\theta}{\theta-1}}}.$$

It is straightforward to see that (3.28) and (3.29) are identical with (3.40) and (3.41), respectively, if they have the an identical $\frac{\partial k_{t+1}}{\partial m_t}$.

For this lump-sum case, it has ambiguous effects of mitigation share m_t on the young-age and the old-age consumption. The explanation of such ambiguity is also similar to the capital income subsidy scheme. Households with lower θ have a stronger consumption smoothing. Such households are very unwilling to reduce their young-age consumption; consequently, they greatly reduce their savings. If the reduction in savings is accompanied by a large increase in interest rate R_{t+1} (that is, when $|\varepsilon_{R,k}|$ is high), the effect of mitigation share m_t on both the young-age consumption c_t^y and the old-age consumption c_{t+1}^o could be positive.

We also derive the effects of $\mathcal{T}_{t+1}$ on the optimal consumption-savings:

$$\frac{\partial k_{t+1}^*}{\partial \mathcal{T}_{t+1}} = -\frac{(\beta R_{t+1})^{\frac{1}{\theta-1}}}{1 + (\beta^{\frac{1}{\theta}} R_{t+1})^{\frac{\theta}{\theta-1}} \left[1 + \frac{\theta}{\theta-1}\varepsilon_{R,k^*}\left(1 + \frac{1}{\theta}\frac{\mathcal{T}_{t+1}}{k_{t+1}^* R_{t+1}}\right)\right]} < 0, \tag{3.42}$$

$$\frac{\partial c_t^y}{\partial \mathcal{T}_{t+1}} = -\frac{\partial k_{t+1}^*}{\partial \mathcal{T}_{t+1}} > 0, \tag{3.43}$$

and

$$\frac{\partial c_{t+1}^o}{\partial \mathcal{T}_{t+1}} = \frac{\partial k_{t+1}^*}{\partial \mathcal{T}_{t+1}} R_{t+1}\left(\varepsilon_{R,k^*} + 1\right) + 1 > 0. \tag{3.44}$$

We obtain different signs for the effects of the lump-sum transfer $\mathcal{T}_{t+1}$ on savings k_{t+1} and the young age consumption c_t^y. The denominator in (3.42) is positive, so it has a negative sign. This effect is unambiguous compared to the partial derivative (3.27) under the capital income subsidy scheme. The insight here is that the capital income subsidy scheme incentivizes the households to increase their savings (if the substitution effect dominates the income effect), while the lump-sum transfer scheme does not. Lump-sum transfers at period $t+1$ acts as a substitute for savings. This argument coincides with the study by Samuelson (1958), where the foreseen lump-sum transfer in the future discourages savings for intertemporal consumption smoothing because the transfer acts as a substitute for the households' savings. The effect of the lump-sum transfer $\mathcal{T}_{t+1}$ on the households' young-age consumption c_t^y mirrrors the partial derivative (3.42), so it has a positive sign.

4 Results and Discussions

4.1 Existence of Pareto-Improving Social Contracts

Pareto-improving social contracts exist if and only if the contracts improve the welfare of at least one generation, either $\mathcal{G}_t$ or $\mathcal{G}_{t+1}$, while leaving the other generation's welfare constant. The welfare for each generation is represented by the lifetime utility (3.1). Let $\mathcal{V}_t^c$ be the lifetime utility of $\mathcal{G}_t$ social contract of capital income subsidy scheme (m_t, τ_{t+1}^o), and let $\mathcal{V}_t^l$ be the utility with lump-sum social contract $(m_t, \mathcal{T}_{t+1})$. The superscripts of the lifetime utilities represent the types of the social contract. Without any social contract, both the mitigation share and the intergenerational transfers, (m_t, τ_{t+1}^o) and $(m_t, \mathcal{T}_{t+1})$, are $(0, 0)$. We denote the lifetime utility under no social contract as $\mathcal{V}_t^0$. The difference in the $\mathcal{G}_t$'s welfare between signing and not signing social contracts is defined as $\Delta\mathcal{V}_t^c = \mathcal{V}_t^c - \mathcal{V}_t^0$ and $\Delta\mathcal{V}_t^l = \mathcal{V}_t^l - \mathcal{V}_t^0$. The social contract should also be accepted by $\mathcal{G}_{t+1}$, and we also denote their lifetime utility gains from signing the contract by $\Delta\mathcal{V}_{t+1}^c$ and $\Delta\mathcal{V}_{t+1}^l$. Therefore, the intergenerational social contract with a capital income subsidy scheme is Pareto-improving if and only if $\Delta\mathcal{V}_t^c \geq 0$ and $\Delta\mathcal{V}_{t+1}^c \geq 0$. Similarly, the Pareto-improving conditions for a lump-sum transfer social contract are $\Delta\mathcal{V}_t^l \geq 0$ and $\Delta\mathcal{V}_{t+1}^l \geq 0$.

We test the existence of Pareto-improving social contracts by constructing indifference curves consisting of combinations of a mitigation share and an intergenerational transfer. Along the curves the agents are indifferent: either signing or not signing the contract. The indifference curve for $\mathcal{G}_t$ is where signing the contract is neither improving nor harming their welfare, that is, $\Delta\mathcal{V}_t^c = 0$ or $\Delta\mathcal{V}_t^l = 0$. Likewise, the indifference curve for $\mathcal{G}_{t+1}$ is either $\Delta\mathcal{V}_{t+1}^c = 0$ or $\Delta\mathcal{V}_{t+1}^l = 0$, depending on which scheme we consider. We then symbolize the indifference curves:

$$\Omega^c \equiv \Delta\mathcal{V}_t^c = 0 \quad \text{and} \quad \psi^c \equiv \Delta\mathcal{V}_{t+1}^c = 0 \quad \text{for a capital income subsidy scheme;}$$

and

$$\Omega^l \equiv \Delta\mathcal{V}_t^l = 0 \quad \text{and} \quad \psi^l \equiv \Delta\mathcal{V}_{t+1}^l = 0 \quad \text{for a lump-sum transfer scheme.}$$

4.1.1 Cobb-Douglas Case

We start our analysis by designating the intertemporal substitution parameter θ and the factors of production substitution parameter ρ to be equal to 0. We utilize limiting arguments because imposing the parameter $\theta = \rho = 0$ into the systems of equations in Table 1 and Table 2 yields divisions by zero. The limiting arguments yield a logarithmic utility function (which is essentially the same as a Cobb-Douglas utility function if we take the natural logarithm of it) and a Cobb-Douglas production function. We put the details of the limiting arguments and the derivations under the Cobb-Douglas case in Appendix A.4 for the capital income subsidy scheme and in Appendix B.3 for the lump-sum transfer scheme.

Table 3: Main Parameter Values

Parameters	Description	Value
t	Starting period of social contract	0
α	Weight of capital	1/3
β	Consumption discount factor	0.7
θ	Consumption substitutabillity parameter	0
ρ	Factors of production substitutabillity parameter	0
δ	Decay rate of the stock pollution	0
γ	Mitigation effort coefficient	1
ξ	Pollution rate of capital stock	1
E_{-1}	Pollution stock in the previous period	0
A	Coefficient for the total productivity factor z	3
k_0	Capital per capita at period 0	2
m_{-1}	Mitigation share from the previous period contract	0
m_1^e	Foreseen mitigation share for the next period contract	0
τ_0^o	Transfer rate for the previous period contract	0
$\tau_2^{o,e}$	Foreseen capital income subsidy rate for the next period contract	0
$\mathcal{T}_0$	Lump-sum transfer for the previous period contract	0
$\mathcal{T}_2^e$	Foreseen lump-sum transfer for the next period contract	0

Table 3 presents the parameters and starting values for the Cobb-Douglas case. The starting period of the social contract $t = 0$ means that the contract is between $\mathcal{G}_0$ and $\mathcal{G}_1$. We follow Dao et al. (2017) for the values of parameters α, β, δ, γ, and ξ. We assign $E_{-1} = 0$ and $A = 3$ to avoid a very small value of total productivity factor $z(E_t)$ in period 0. The capital per capita $k_0 = 2$ is our starting guess, and we will try to vary it as we proceed with our analysis. The foreseen social contract mitigation share m_1^e, the transfer rate of capital income subsidy $\tau_2^{o,e}$, and the lump-sum transfer $\mathcal{T}_2^e$ represent the succeeding contract. We remind that the system of equations in Table 1 and Table 2 relate to $\mathcal{G}_t$. The system of equations for $\mathcal{G}_{t+1}$ is similar except that the equations are shifted by one period.[12] We assume there are no preceding and succeeding social contracts, both for capital income

[12] For example, the young-age consumption for $\mathcal{G}_{t+1}$ under the capital income subsidy scheme is the equation (3.15) shifted forward by one period:

$$c_{t+1}^y = \Big(1 - s(R_{t+2}^e, \tau_{t+2}^{o,e})\Big) I_{t+1}(1 - m_{t+1}^e).$$

subsidy and lump-sum scheme, so there is no mitigations nor transfers in the previous and the next period.

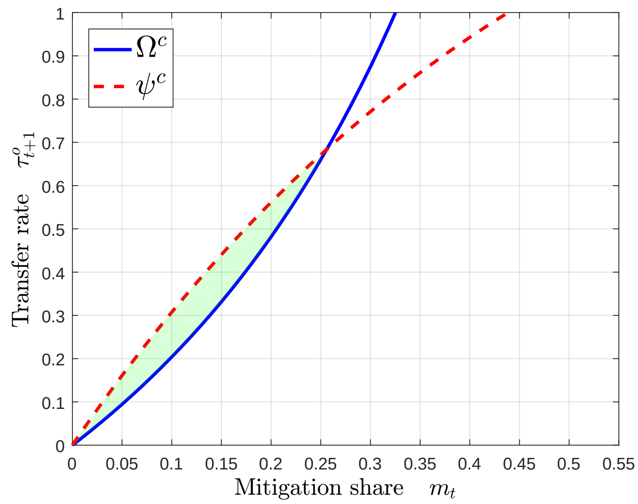


Figure 5: Capital Income Subsidy Scheme with $\theta = 0$, $\rho = 0$, and $k_0 = 2$

We construct indifference curves for capital income subsidy scheme in Figure 5. The solid-line curve Ω^c represents the intergenerational social contracts under which the households $\mathcal{G}_t$ is indifferent between accepting and not accepting the contract. Likewise, the dashed-line curve ψ^c represents the combinations of contracts for households $\mathcal{G}_{t+1}$ to be indifferent. The area above Ω^c illustrates the sets of contracts (m_t, τ^o_{t+1}) where it improves the welfare for households $\mathcal{G}_t$. For households $\mathcal{G}_{t+1}$, the welfare-improving set of contracts lies in the area under the curve ψ^c. Pareto-improving social contracts exist if there exist an area where the dashed-line curve is above the solid-line curve. We call this area as a Pareto-improving set of social contracts $\mathcal{P}^c$.[13] Intuitively, households $\mathcal{G}_t$ gains more from the contract if they receive a higher capital income subsidy τ^o_{t+1} while holding the mitigation share m_t constant. Conversely, $\mathcal{G}_{t+1}$ would be better-off with a lower transfer rate τ^o_{t+1} while holding the mitigation share m_t constant. Based on Figure 5, we can infer that a Pareto-improving set of social contracts exist, denoted by the shaded area. Households $\mathcal{G}_t$ and $\mathcal{G}_{t+1}$ could bargain to determine an appropriate contract (m^*_t, τ^{o*}_{t+1}) inside the $\mathcal{P}^c$ area.

We also analyze the existence of $\mathcal{P}^c$ with a lower value of k_0. Dao et al. (2017) argue that $\mathcal{P}^c$ ceases to exist if the income is below a certain income threshold. Because we assume the exogenous variable is capital per capita k_t, we derive the threshold for capital per capita:

$$\hat{k}_t = \left(\frac{1}{(1-\alpha) z(E_{t-1})\,(1-\tau^y_t)} \frac{\alpha(1+\alpha\beta)(1+\beta)^2}{(1-\alpha)(\beta+\gamma+\gamma\beta)\beta^2} \right)^{\frac{1}{\alpha}}. \tag{4.1}$$

The details of the threshold derivations are in Appendix A.4, particularly on expressions

[13] The explanation of why such an area is Pareto-improving has been explained by Dao et al. (2017). We reproduce their derivations in the Appendix A.4, particularly see expressions (A.46) and (A.47) regarding the Pareto-improvement conditions.

(A.52). Using the parameters shown in Table 3, the capital per capita threshold is approximately $\hat{k}_0 \approx 0.435$.

Figure 6 displays the indifference curves with a capital per capita below the threshold. The curve ψ^c is rotated downward, and this causes the area $\mathcal{P}^c$ to be nonexistent. With our assumption that the mitigation effort is carried out as a share of labor income w_t, we can infer that the mitigation effort becomes higher with higher labor income. The labor income is also positively affected by the initial capital per capita k_0. With higher k_0, households $\mathcal{G}_{t+1}$ would accept a contract with the same mitigation share m_t at a higher transfer rate τ^o_{t+1}, and therefore rotates the curve ψ^c counterclockwise.[14]

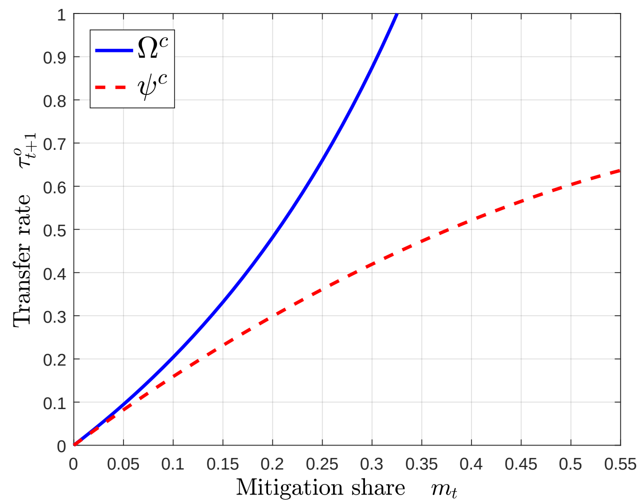


Figure 6: Capital Income Subsidy Scheme with $\theta = 0$, $\rho = 0$, and $k_0 = 0.4$

Next, we test the existence of Pareto-improving social contracts for a lump-sum transfer scheme. Figure 7 has the same parameter values as in Figure 6. This lump-sum transfer scheme yields a Pareto-improving sets of social contracts, where we denote it by $\mathcal{P}^l$, even though the initial capital per capita k_0 is set to be below the threshold $\hat{k}_t$. We cannot derive the threshold $\hat{k}_t$ for the lump-sum scheme because its system of equations has nonlinear simultaneous equations for k_{t+1} and R_{t+1}. Instead, we simulate it with lower values of k_0 and search when the Pareto-improving sets of contracts are nonexistent $\mathcal{P}^l = \varnothing$.

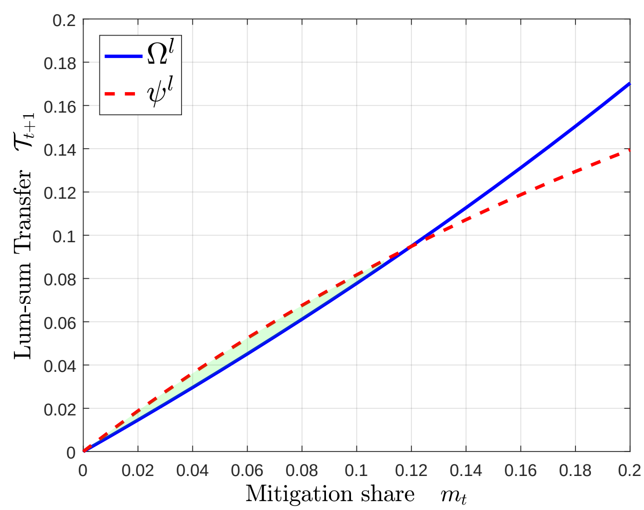


Figure 7: Lump-sum Transfer Scheme with $\theta = 0$, $\rho = 0$, and $k_0 = 0.4$

[14]See the last expression of (A.47) to see the relationship between I_t and ψ^c mathematically.

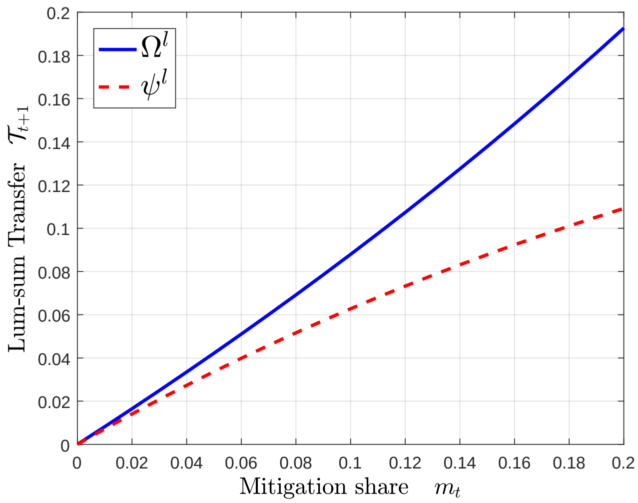
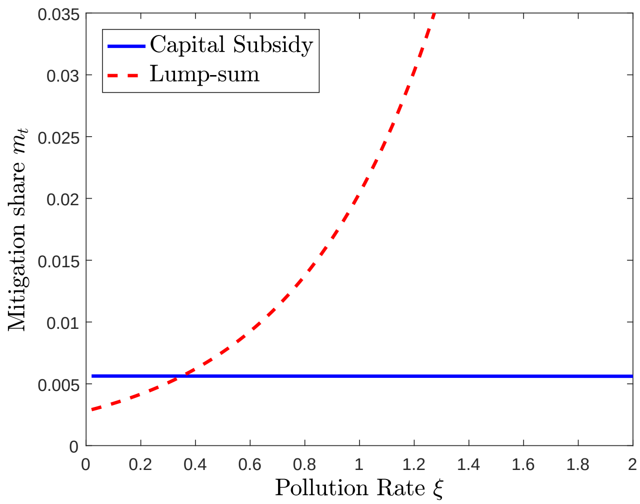


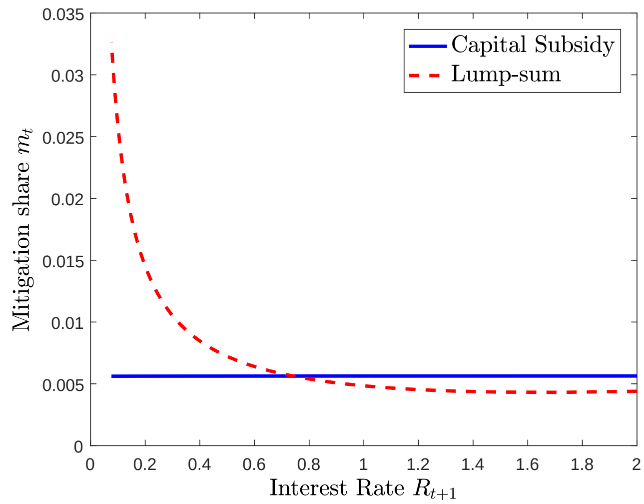
Figure 8: Lump-sum Transfer Scheme with $\theta = 0$, $\rho = 0$, and $k_0 = 0.2$

We present this case in Figure 8 with $k_0 = 0.2$. We could stipulate that imposing this calibration, lump-sum transfer scheme requires lower threshold of k_0 compared to capital income subsidy scheme. This seems to be one of the advantages of lump-sum transfer scheme. Comparing Figure 7 and Figure 8, the lower value of k_0 rotates the dashed-line downward while rotates the solid line upward. The lump-sum transfer scheme has this effect because the indifference curve for households $\mathcal{G}_t$ is affected by the value of the savings.

We could refer to the comparative-static analysis, particularly expressions (3.27) and (3.42). Under Cobb-Douglas Case ($\theta = 0$), an increase in capital subsidy τ^o_{t+1} does not change the savings k_{t+1}, whereas an increase in a lump-sum transfer $\mathcal{T}_{t+1}$ discourages savings. A reduction in savings has two advantages: it reduces the externality caused by capital accumulation, and it reduces a dynamic inefficiency when the net marginal return on capital is lower than the population growth rate. The first advantage originates from our model specification regarding the pollution stock equation (3.9). Savings increase capital accumulation which generates negative externality; therefore, a decrease in savings also decreases the externality. The second advantage is related to the golden rule of savings, in which steady-state consumption is maximized when the net marginal return on capital is equal to the population growth rate. We denote interest rate $R_t = 1 + r_t$. Dynamic efficiency occurs if r_t equals population growth rate (Heijdra, 2009, p. 905-906). In our model, the population growth rate is zero; hence, the dynamic efficiency is when $R_{t+1} = 1$. Households $\mathcal{G}_t$ oversave when $R_{t+1} < 1$ and undersave if $R_{t+1} > 1$, indicating a dynamic inefficiency. The lump-sum transfer scheme is therefore welfare-improving for households $\mathcal{G}_t$ if they oversave, that is, when the parameterization yields a low interest rate R_{t+1} (before introducing a social contract).



(a) Capital Externality Simulations with $\theta = 0$, $\rho = 0$, $R_{t+1} = 0.13$, $\tau^o_{t+1} = 0.01$, and $\mathcal{T}_{t+1} = 0.005$



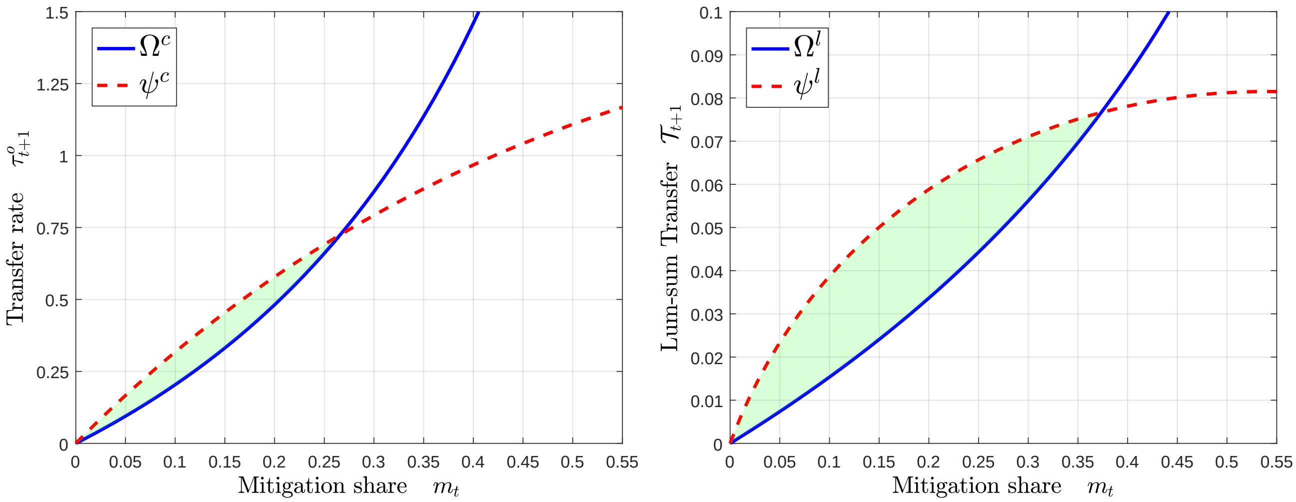
(b) Interest Rate Simulations with $\theta = 0$, $\rho = 0$, $\xi = 1$, $\tau^o_{t+1} = 0.01$, and $\mathcal{T}_{t+1} = 0.005$

Figure 9: Scheme Comparisons with Different Calibrations

We plot indifference curves for households $\mathcal{G}_t$ along which their utility is constant, given the combinations of transfers (either lump-sum $\mathcal{T}_{t+1}$ or capital subsidy τ^o_{t+1}) and the mitigation shares m_t. We construct the indifference curves in Figure 9 by varying pollution rate ξ and interest rate R_{t+1}. The transfers are chosen such that neither scheme is very superior compared to the other scheme due to a very high transfer. A Higher mitigation share m_t in Figure 9 means that households $\mathcal{G}_t$ are willing to perform a higher mitigations given the same transfers. Therefore, a higher mitigation share m_t may have a larger Pareto-improving set of social contracts ($\mathcal{P}$-set).

Figure 9a confirms the first advantage of a lump-sum transfer scheme: the reduction of the capital accumulation externality. This advantage is more prominent as the pollution rate ξ increases. The lump-sum transfer scheme loses its superiority if discouraging savings is less beneficial, that is, when the pollution rate is low. Reducing savings is also unfavourable if the interest rate R_{t+1} is high compared to the population growth rate. This argument is supported by Figure 9b. Because we employ endogenous interest rate, we construct Figure 9b by simulating various levels of capital per capita k_0. The lump-sum transfer scheme is superior with a low interest rate R_{t+1}. As interest rates increase, the mitigation share m_t under lump-sum scheme becomes lower than the capital income subsidy scheme.

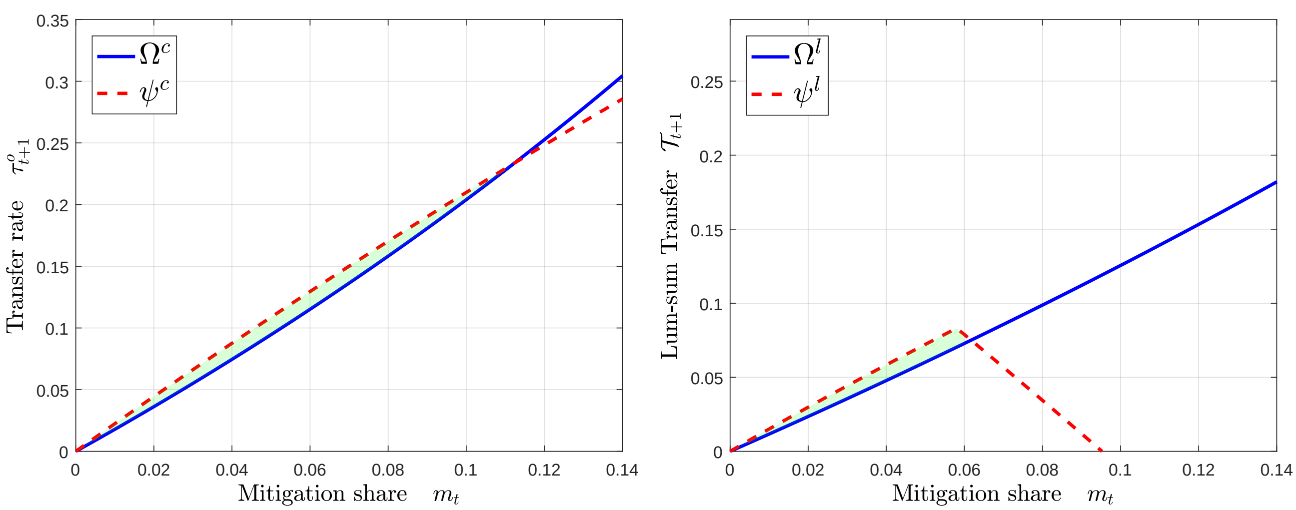
Figure 10 shows that the lump-sum transfer scheme has a higher Pareto-improving contracts feasibility under a high pollution rate, while Figure 11 shows that the capital income subsidy scheme is superior under a low pollution rate. The lump-sum transfer and the capital income subsidy scheme have different vertical axes. Nevertheless, we can compare their $\mathcal{P}$-set by examining the mitigation share m_t where the the two curves, Ω and ψ, cross. This point implies the highest possible mitigation share based on the $\mathcal{P}$-set. The lump-sum transfer scheme has a higher possible mitigation share than the capital income subsidy scheme if the pollution rate is high. This conforms with our argument that a larger decrease in savings, caused by choosing a lump-sum transfer scheme in a Cobb-Douglas case, is more advantageous for a high capital accumulation externality.



(a) Capital Income Subsidy Scheme
with $\theta = 0$, $\rho = 0$, $k_0 = 2$, and $\xi = 1.1$

(b) Lump-sum Transfer Scheme
with $\theta = 0$, $\rho = 0$, $k_0 = 2$, and $\xi = 1.1$

Figure 10: Scheme Comparison with a High Pollution Rate

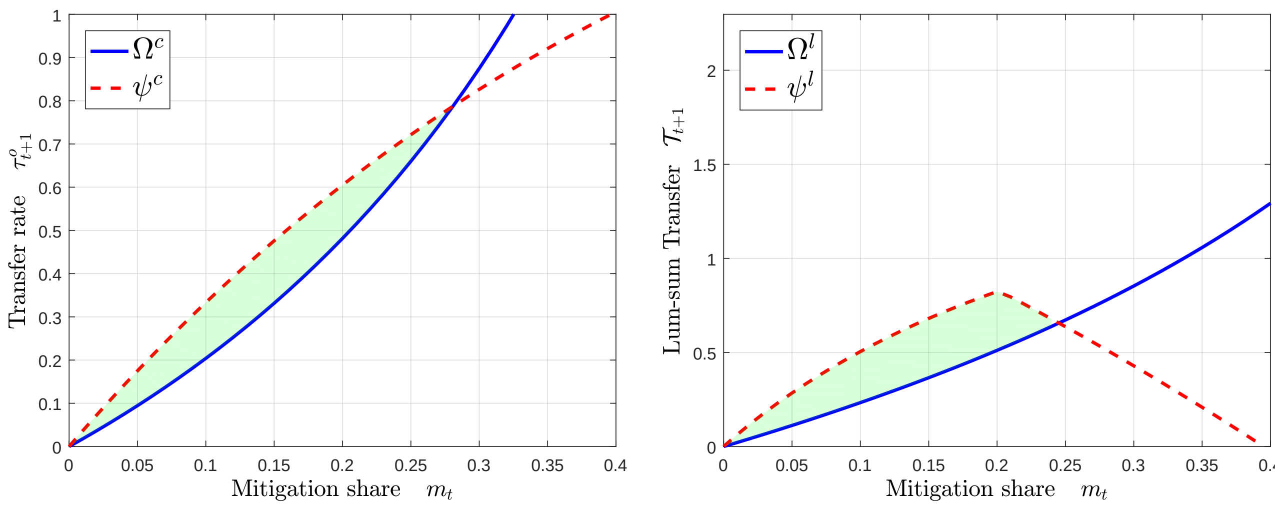


(a) Capital Income Subsidy Scheme
with $\theta = 0$, $\rho = 0$, $k_0 = 2$, and $\xi = 0.05$

(b) Lump-sum Transfer Scheme
with $\theta = 0$, $\rho = 0$, $k_0 = 2$, and $\xi = 0.05$

Figure 11: Scheme Comparison with a Low Pollution Rate

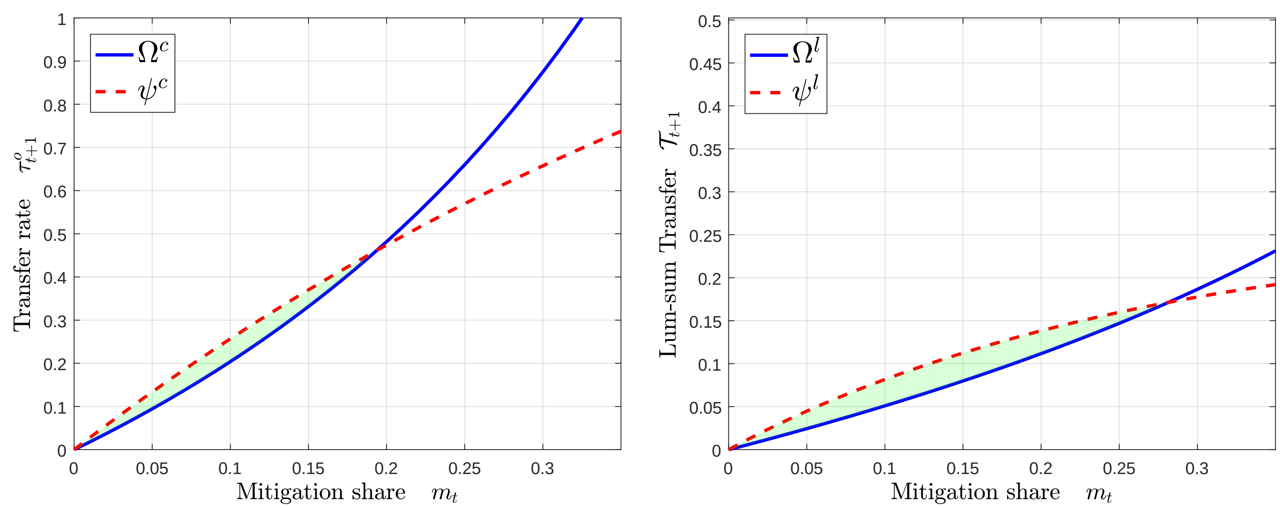
We also test the existence of $\mathcal{P}$-set for different interest rates R_{t+1}. We calibrate the parameter values such that it yields both a high and a low interest rate before introducing a social contract, as shown in Figure 12 and Figure 13, respectively. The results of the $\mathcal{P}$-set areas show that the lump-sum transfer scheme is superior if such a scheme is introduced in a dynamically inefficient economy with $R_{t+1} < 1$. On the contrary, the capital income subsidy scheme has a higher mitigation share in the $\mathcal{P}$-set if the interest rate is high.



(a) Capital Income Subsidy Scheme with $\theta = 0$, $\rho = 0$, $k_0 = 0.5$, $A = 6$, $\xi = 0.5$, and $R_{t+1} = 1.3$

(b) Lump-sum Transfer Scheme with $\theta = 0$, $\rho = 0$, $k_0 = 0.5$, $A = 6$, $\xi = 0.5$, and $R_{t+1} = 1.3$

Figure 12: Scheme Comparison with a High Interest Rate R_{t+1}



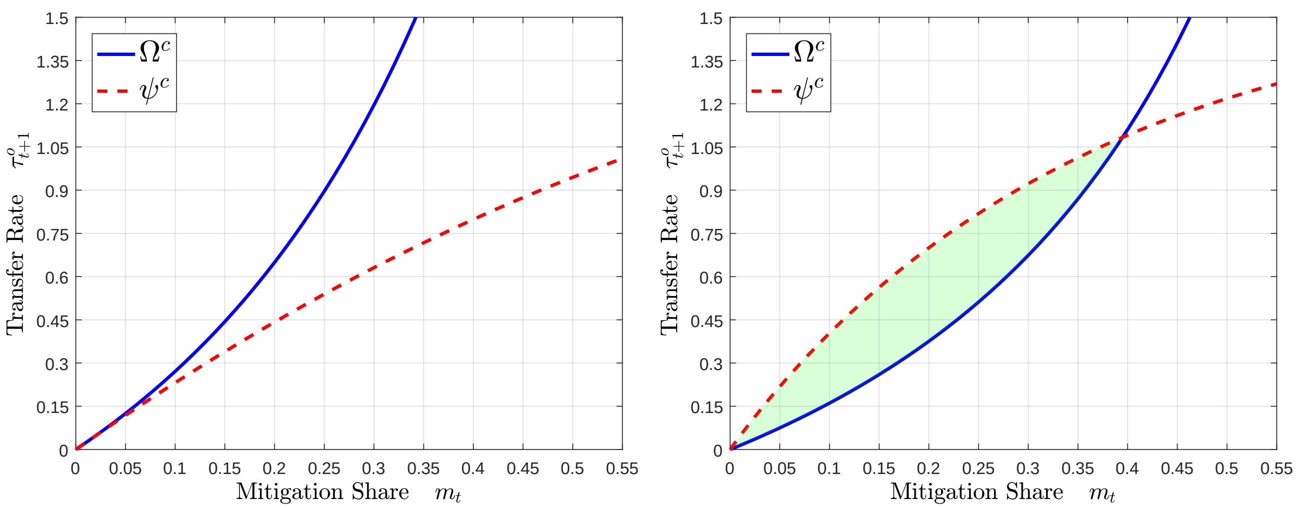
(a) Capital Income Subsidy Scheme with $\theta = 0$, $\rho = 0$, $k_0 = 2$, $A = 3$, $\xi = 0.5$, and $R_{t+1} = 0.359$

(b) Lump-sum Transfer Scheme with $\theta = 0$, $\rho = 0$, $k_0 = 2$, $A = 3$, $\xi = 0.5$, and $R_{t+1} = 0.359$

Figure 13: Scheme Comparison with a Low Interest Rate R_{t+1}

4.1.2 Intertemporal Consumption Substitutability

In this section, we evaluate the indifference curves Ω and ψ with different degrees of intertemporal consumption substitutability θ. To test the existence of Pareto-improving social contracts $\mathcal{P}$, we employ the same approach as in the Cobb-Douglas case section 4.1.1. The area above Ω's curve represents welfare-improving sets contracts for households $\mathcal{G}_t$, while the area below the ψ's curve represents welfare-improving contracts for households $\mathcal{G}_{t+1}$. The area between the curves characterizes welfare-improving contracts for both $\mathcal{G}_t$ and $\mathcal{G}_{t+1}$: proving an existence of $\mathcal{P}$. Main parameters in Table 3 is still chosen as the base values. When we use a different parameterization, we put the changed parameters below the figure.



(a) Easy substitution with $\theta = 0.1$, $\rho = 0$, $k_0 = 2$, $A = 3$, $\xi = 1$, and $R_{t+1} = 0.147$

(b) Difficult substitution with $\theta = -0.1$, $\rho = 0$, $k_0 = 2$, $A = 3$, $\xi = 1$, and $R_{t+1} = 0.121$

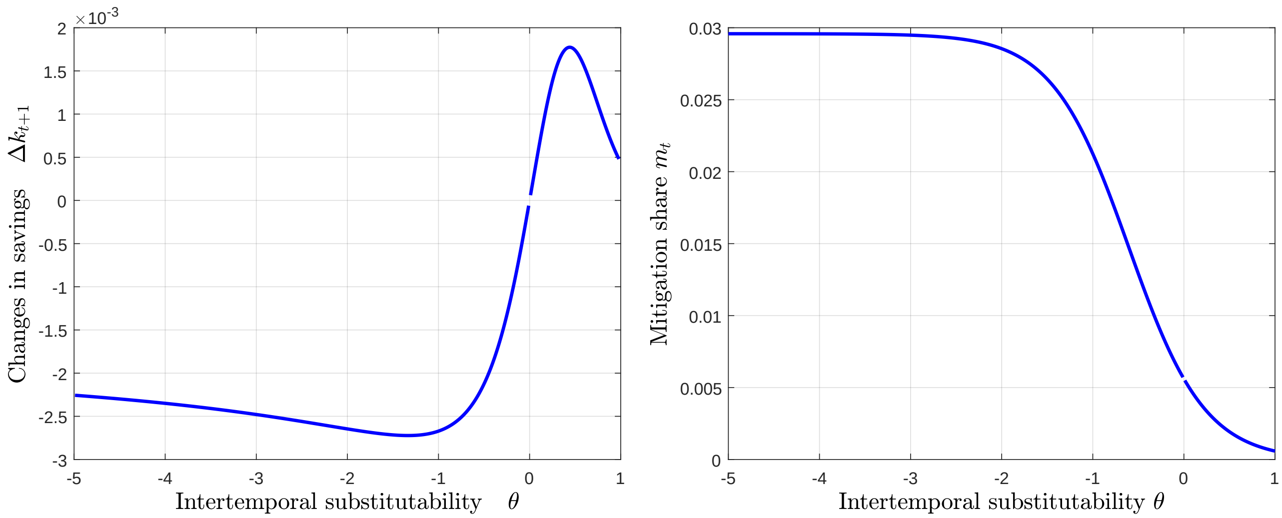
Figure 14: Capital Income Subsidy Scheme with Different Consumption Substitutabilities

Let us test the existence of $\mathcal{P}^c$, Pareto-improving contracts for capital income subsidy scheme. The left side of Figure 14 represents an easy intertemporal consumption substitution (relative to the logarithmic utility case), whereas the right-side represents a difficult intertemporal consumption substitution. A smaller value of the parameter θ improves the feasibility of intergenerational social contracts. The case with easy substitution barely has an area of $\mathcal{P}^c$, while the difficult substitution case has a relatively wide area of $\mathcal{P}^c$. In comparison with Figure 5 which has a highest possible mitigation share in the $\mathcal{P}^c$-set around 0.25, Figure 14 has a higher value around 0.38. This conjecture also holds with different values of parameter ρ.[15] We also show the endogenous interest rate R_{t+1} under no social contract below each figure. The interest rate differs because a different parameter

[15]See Appendix C.1 for the cases with different factors of production substitution parameter ρ.

θ yields a different interest rate.

To understand the effects of parameter θ mathematically, we refer back to the comparative-static analysis equations (3.24) and (3.27). For a low value of θ, households are more inequality averse regarding their intertemporal consumption. For these households, an increase in mitigation share m_t greatly reduce their savings, by more than for households who care less of the inequality in their intertemporal consumption. An increase in the capital income subsidy τ^o_{t+1} also lowers savings when $\theta < 0$, while an increase in τ^o_{t+1} raises savings if $\theta > 0$. A greater reduction in savings is welfare-improving with a low interest rate (relative to population growth rate) and a high pollution rate. The parameters choosen for constructing Figure 14 results in a low interest rate before any contract is introduced. We also impose the pollution rate $\xi = 1$ for Figure 14, which is relatively high.



(a) Changes in savings with $\rho = 0$, and $\tau^o_{t+1} = 0.01$

(b) Mitigation share with $\rho = 0$ and $\tau^o_{t+1} = 0.01$

Figure 15: Intertemporal Consumption Substitutability Simulations for Capital Income Subsidy Scheme

We present the dynamics of the intertemporal consumption substitutability θ due to an initiation of a capital subsidy transfer τ^o_{t+1} in Figure 15. The left side of the figure illustrates how the households change their savings if they expect a capital subsidy τ^o_{t+1}. This figure agrees with our mathematical interpretations that households $\mathcal{G}_t$ reduce their savings with a low value of θ. The right-side of Figure 15 illustrates indifference curves reflecting the maximum amount of mitigation share m_t for the households $\mathcal{G}_t$ provided that they will receive a certain capital subsidy τ^o_{t+1}. Figure 15b shows that a smaller θ increases the Pareto-improving contracts $\mathcal{P}^c$ feasibility.

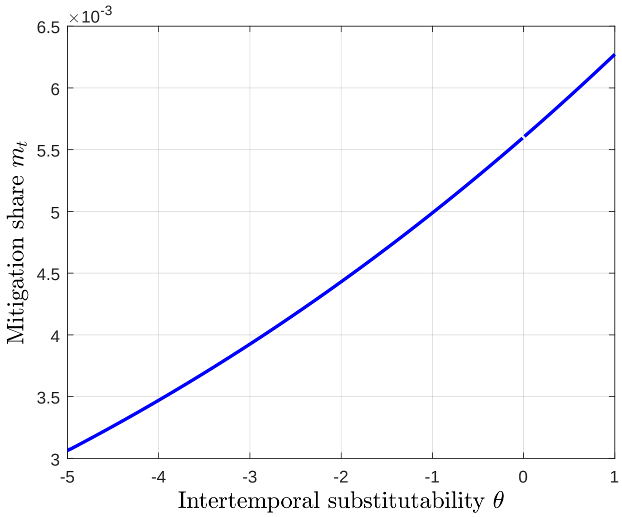
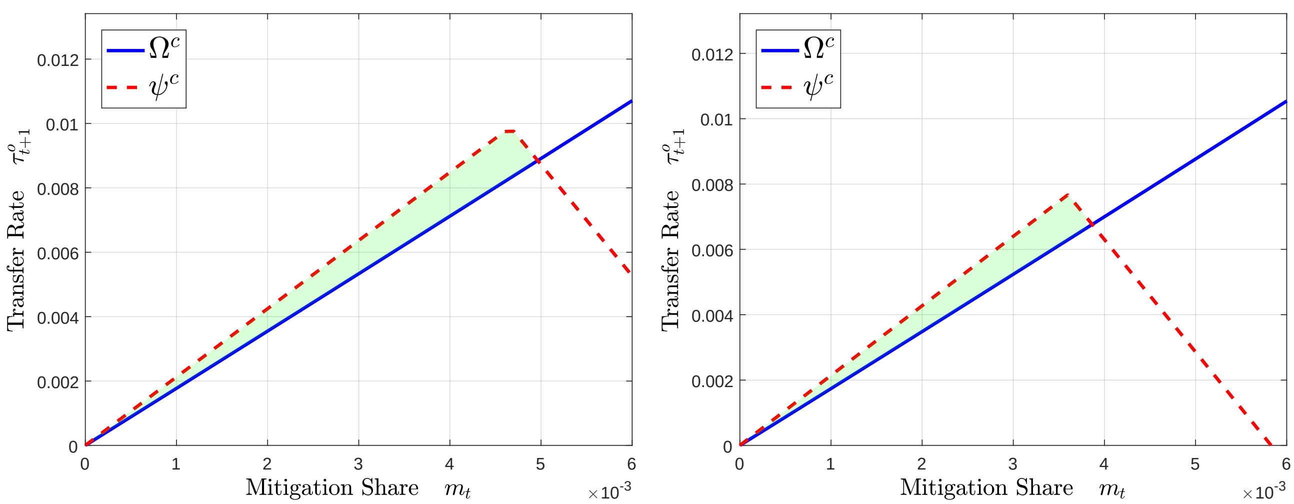


Figure 16: Intertemporal Substitutability Simulations for Capital Income Subsidy Scheme with $\rho = 0$, $\xi = 0.05$, $A = 6$ and $k_0 = 0.5$

However, discouraging savings is beneficial when the interest rate R_{t+1} is low and the pollution rate ξ is high. This is what Figure 16 tells us. Figure 16 is a recalibration of Figure 15b with a lower pollution rate ξ and a relatively higher interest rate (by imposing a lower initial capital per capita k_0). We also adjust the productivity coefficient A such that the total production Y_t remains stable. Under this parameterization, the intertemporal consumption substitutability θ has a positive relationship with the mitigation share m_t. This implies that, in contrast to Figure 15b, smaller θ may decrease the Pareto-improving contracts $\mathcal{P}^c$ feasibility.



(a) Easy substitution with $\theta = 0.1$, $\rho = 0$, $k_0 = 0.1$, $A = 8$, $R_{t+1} = 3.1$, and $\xi = 0.01$

(b) Difficult substitution with $\theta = -1$, $\rho = 0$, $k_0 = 0.1$, $A = 8$, $R_{t+1} = 2.6$, and $\xi = 0.01$

Figure 17: Capital Income Subsidy Scheme with Different Consumption Substitutabilities

We test the existence of Pareto-improving $\mathcal{P}^c$ set of social contracts with a low pollution rate ξ and a high interest rate R_{t+1} in Figure 17. This figure supports our reasoning that a lower θ shrinks the $\mathcal{P}^c$-set if lowering savings is undesirable, that is, when the capital accumulation externality is low and the interest rate is high.

The discussions regarding the feasibility of $\mathcal{P}^c$-set carry over to the $\mathcal{P}^l$-set for the lump-sum transfer scheme. The key factor here is to examine whether a reduction in savings is beneficial. We have explained in the comparative-static analysis the difference between the effect of lump-sum transfers $\mathcal{T}_{t+1}$ and capital subsidy rate τ^o_{t+1}. An expected lump-sum transfer $\mathcal{T}_{t+1}$ discourages savings, regardless of the value of the intertemporal consumption substitutability θ. An increase in the capital subsidy rate τ^o_{t+1}, on the other hand, reduces savings if only if $\theta < 0$. We show this difference in Figure 18. Lowering θ yields a greater reduction in savings for the capital income subsidy scheme than for the lump-sum transfer scheme.

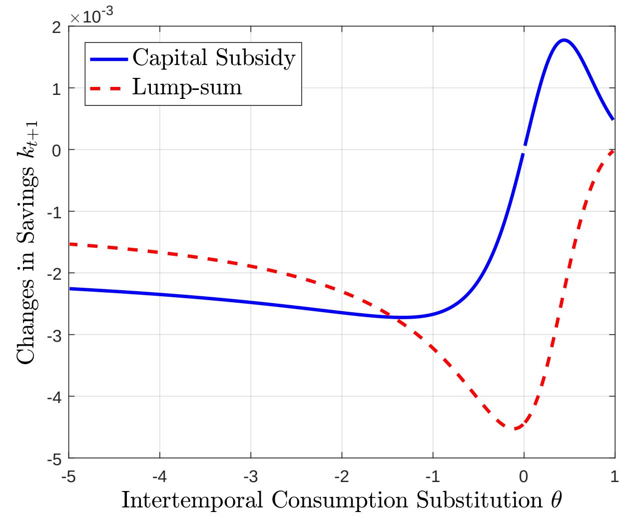
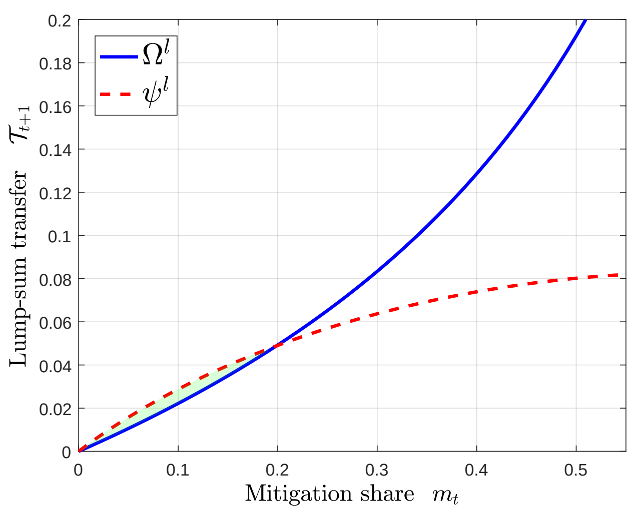
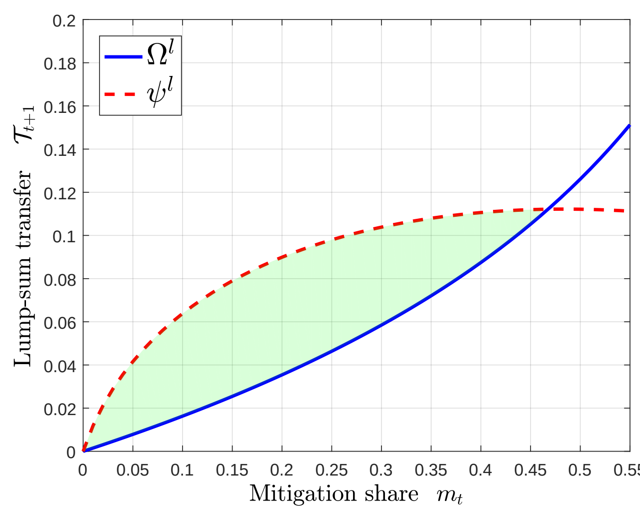


Figure 18: Intertemporal Substitutability Simulations for both Schemes with $\rho = 0$, $\xi = 1$, $k_0 = 2$, $A = 3$, $\tau^o_{t+1} = 0.01$, and $\mathcal{T}_{t+1} = 0.001$

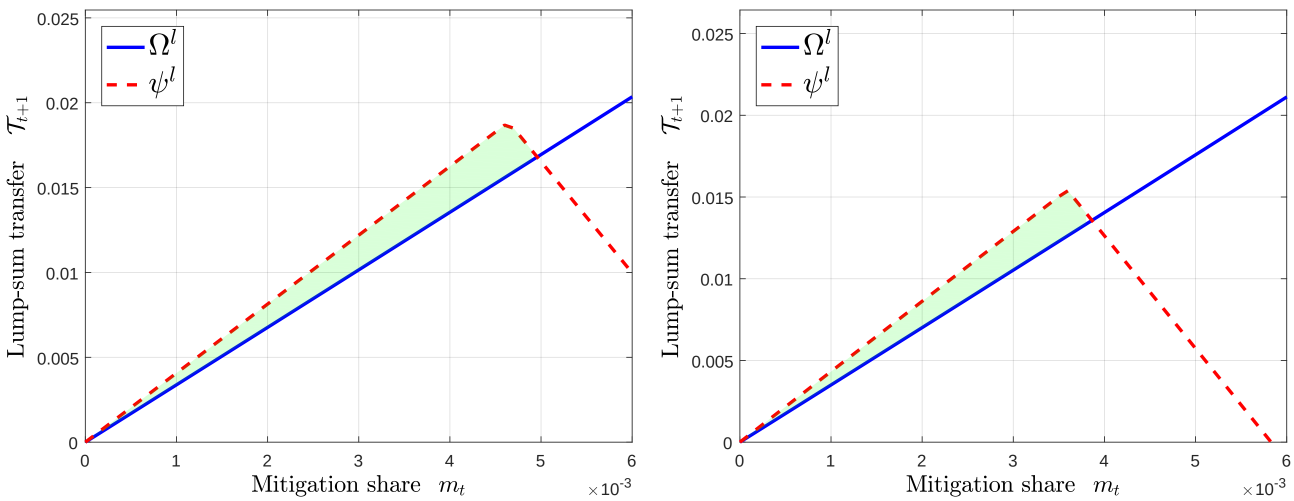


(a) Easy substitution with $\theta = 0.1$, $\rho = 0$, $k_0 = 2$, $A = 3$, $\xi = 1$, and $R_{t+1} = 0.147$



(b) Difficult substitution with $\theta = -0.1$, $\rho = 0$, $k_0 = 2$, $A = 3$, $\xi = 1$, and $R_{t+1} = 0.121$

Figure 19: Lump-sum Transfer Scheme with Different Consumption Substitutabilities



(a) Easy substitution with $\theta = 0.1$, $\rho = 0$, $k_0 = 0.1$, $A = 8$, $R_{t+1} = 3.1$, and $\xi = 0.01$

(b) Difficult substitution with $\theta = -1$, $\rho = 0$, $k_0 = 0.1$, $A = 8$, $R_{t+1} = 2.6$, and $\xi = 0.01$

Figure 20: Lump-sum Transfer Scheme with Different Consumption Substitutabilities

We proceed to analyze the existence of Pareto-improving contracts $\mathcal{P}^l$ with different intertemporal consumption substitutabilities. Figure 19 is calibrated for the case when reducing savings is favorable. In Figure 20, we impose the same parameter values as in Figure 17, that is, when a decrease in savings is less beneficial. Overall, we have similar results as in the capital income subsidy scheme: (1) lowering the intertemporal consumption substitutability θ *enlarges* the $\mathcal{P}$-set with a low interest rate R_{t+1} and a high pollution rate ξ; but, (2) lowering the intertemporal elasticity of substitution θ *shrinks* the $\mathcal{P}$-set with a high interest rate R_{t+1} and a low the pollution rate ξ. In the case with a high interest rate and a high pollution rate, the total effect (whether the $\mathcal{P}$-set area shrinks or expands) depends on which effects dominates. Reducing savings decrease a high negative externality if the pollution rate is high, but it introduces dynamic inefficiency to the economy because of the high interest rate. The contradictory effects are also true for a low interest rate and a low pollution rate.

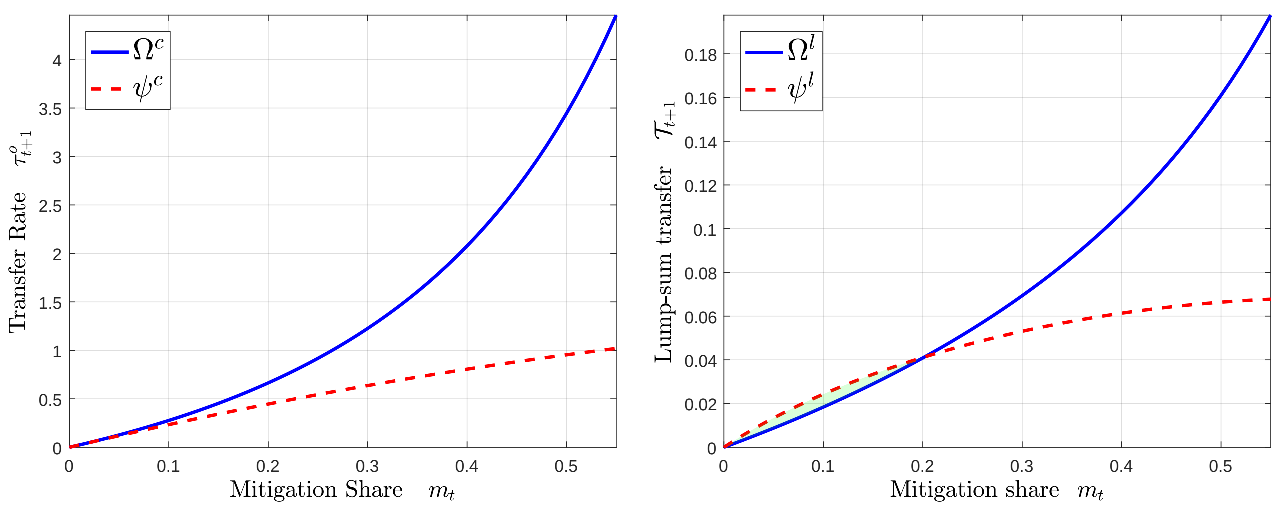
The intuition behind those results is as follows. Establishing a social contract changes consumption-savings decisions. Households who are highly reluctant to substitute their intertemporal consumption will greatly reduce their savings to dampen the changes. This savings reduction is more beneficial for the households provided that the economy initially has a low interest rate (relative to the population growth rate) and a high pollution rate. This is because the reduction in savings will increase the interest rate, so the households receive a higher net return due to the capital crowding-out. A decrease in savings also reduce the capital accumulation externality, which is welfare-improving for the households.

Therefore, Pareto-improving social contracts are more feasible to be established for households who are stricter on their intertemporal consumption provided that reducing savings is beneficial.

4.2 Capital Income Subsidy Versus Lump-Sum Transfer Scheme

Our discussions so far have suggested that the pros and cons of the two schemes hinge on how the schemes affect savings (and by how much). The assumption about the pollution rate ξ and the calibrations of interest rate R_{t+1} then determines which shceme is better in terms of its Pareto-improving feasibility. In the Cobb-Douglas case, we have shown that a capital income subsidy scheme yields a less reduction in savings.[16] Therefore, under the Cobb-Douglas case, lump-sum transfer is superior if the pollution rate ξ is high and the interest rate R_{t+1} is low.

We analyze the existence of Pareto-improving social contracts $\mathcal{P}$ with different intertemporal consumption substitutabilities θ. We are particularly interested in how the different favorability in savings reduction affects $\mathcal{P}$-set for both capital income subsidy scheme and lump-sum transfer scheme. First, let us consider the case of an easy intertemporal substitution case, a high pollution rate ξ, and a low interest rate R_{t+1}. This case is illustrated in Figure 21.



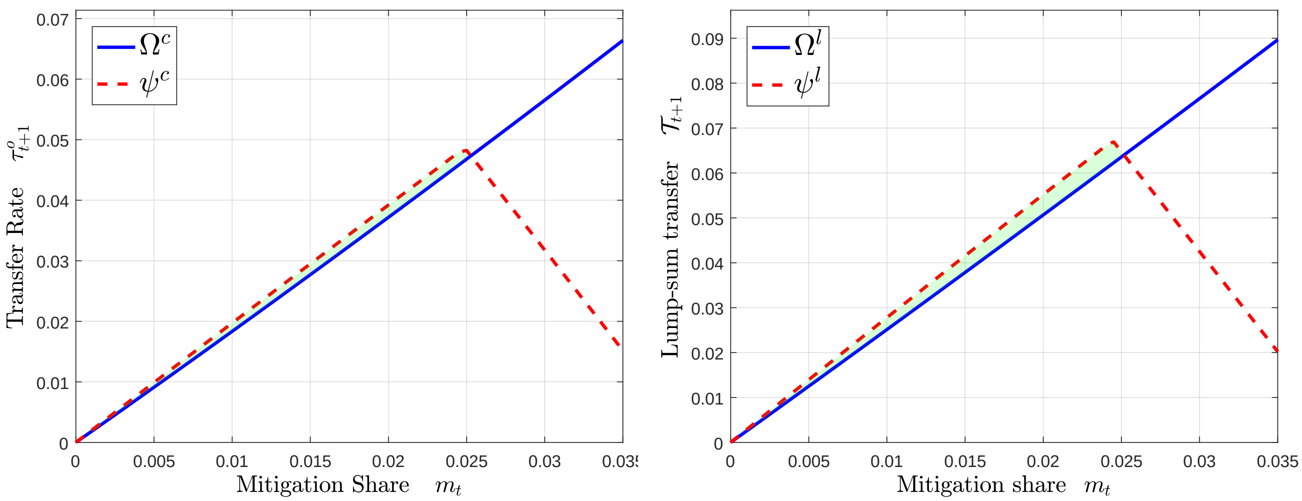
(a) Capital income subsidy scheme with $\theta = 0.1$, $\rho = 0$, $A = 3$, $\xi = 1.1$, $R_{t+1} = 0.12$, and $k_0 = 2$

(b) Lump-sum transfer scheme with $\theta = 0.1$, $\rho = 0$, $A = 3$, $\xi = 1.1$, $R_{t+1} = 0.12$, and $k_0 = 2$

Figure 21: Comparison of Schemes when Savings Reduction is Beneficial

[16] See Figure 18 or the comparative-static analysis. An increase in the capital subsidy rate τ^o_{t+1} does not affect savings when $\theta = 0$, while a lump-sum transfer $\mathcal{T}_{t+1}$ does.

The $\mathcal{P}$-set for lump-sum transfer scheme is larger than the capital income subsidy scheme in Figure 21 because the parameters are choosen such that discouraging savings is beneficial. We have explained in the comparative-static analysis that a capital subsidy τ^o_{t+1} increase households' savings when θ is positive. Lump sum transfer scheme yield a higher decrease savings, so lump-sum transfer is superior under this case.

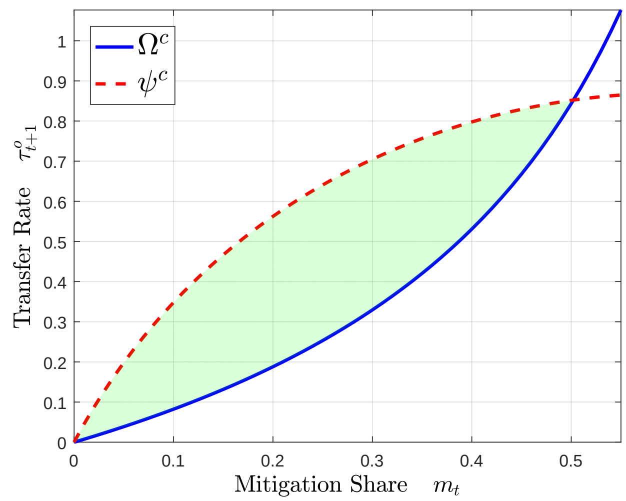


(a) Capital income subsidy scheme with $\theta = 0.1$, $\rho = 0$, $A = 6$, $\xi = 0.05$, $R_{t+1} = 2$, and $k_0 = 0.2$

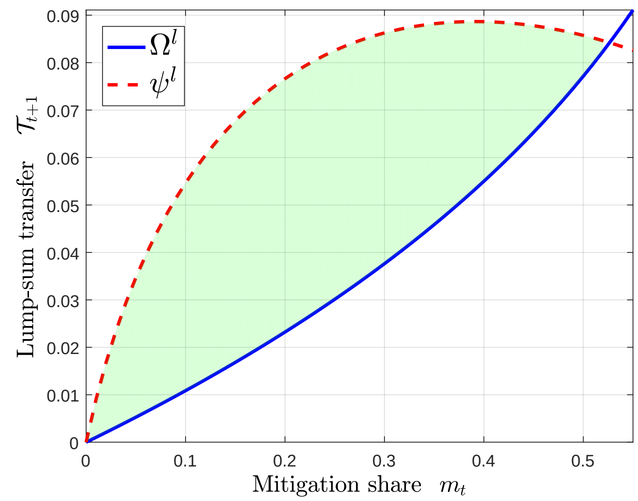
(b) Lump-sum transfer scheme with $\theta = 0.1$, $\rho = 0$, $A = 6$, $\xi = 0.05$, $R_{t+1} = 2$, and $k_0 = 0.2$

Figure 22: Comparison of Schemes when Savings Reduction is Less Beneficial

Next, we consider the case when reducing savings is unfavorable by imposing a lower pollution rate and calibrating a higher endogenous interest rate. Figure 22 presents this case. The lump-sum transfer scheme seems to lose its superiority because now it has a similar maximum mitigation share in its $\mathcal{P}$-set (around 0.025) compared to the capital income subsidy scheme. These findings support our argument that the initial conditions of the economy influence whether a decrease in savings caused by the social contracts is beneficial.

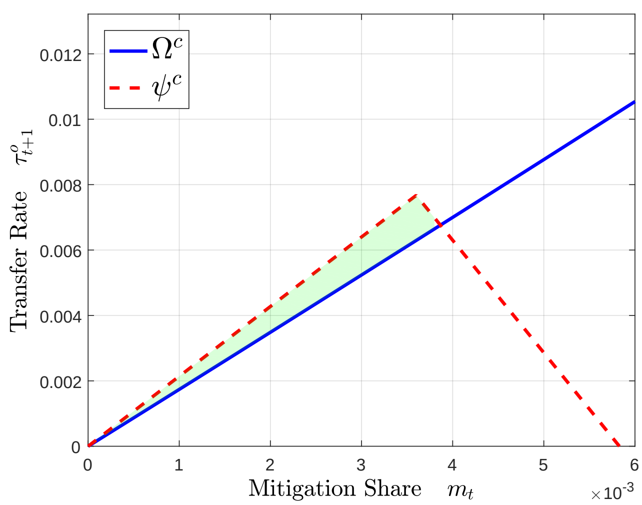


(a) Capital income subsidy scheme with $\theta = -1$, $\rho = 0$, $A = 1.5$, $\xi = 1$, $R_{t+1} = 0.25$, and $k_0 = 1$

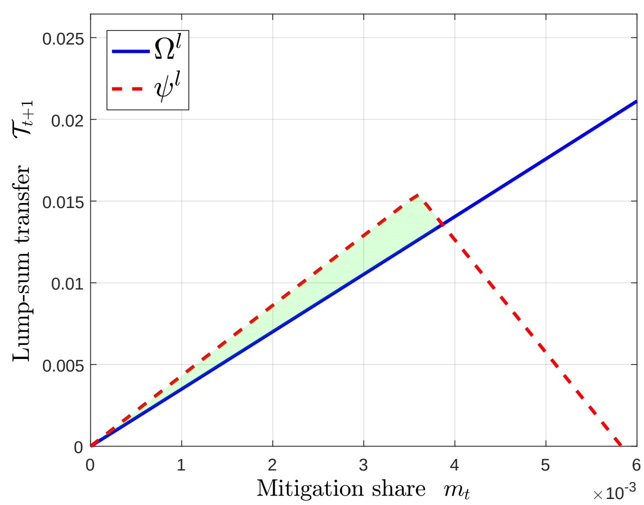


(b) Lump-sum transfer scheme with $\theta = -1$, $\rho = 0$, $A = 1.5$, $\xi = 1$, $R_{t+1} = 0.25$, and $k_0 = 1$

Figure 23: Comparison of Schemes when Savings Reduction is Beneficial



(a) Capital income subsidy scheme with $\theta = -1$, $\rho = 0$, $A = 8$, $\xi = 0.01$, $R_{t+1} = 3.1$, and $k_0 = 0.1$



(b) Lump-sum transfer scheme with $\theta = -1$, $\rho = 0$, $A = 8$, $\xi = 0.01$, $R_{t+1} = 3.1$, and $k_0 = 0.1$

Figure 24: Comparison of Schemes when Savings Reduction is Less Beneficial

We also graph the $\mathcal{P}$-set for a difficult substitution case. Both a favorable and an unfavorable initial conditions for a decrease in savings (relatively based on the interest rate R_{t+1} and pollution rate ξ) is presented in Figure 23 and Figure 24, respectively. The results are similar to the easy substitution case when θ is positive. The lump-sum transfer scheme is superior when a decrease in savings is favorable (Figure 23b), but it is not the case when a decrease in savings is unfavorable (Figure 24b). To understand why we have similar results with the easy substitution case, we compare the degree of changes in savings for both schemes, with the current calibrations.

We recalibrate Figure 18 using the parameter values of Figure 23. Figure 25 shows that with the parameter $\theta = -1$, lump-sum transfer scheme yields a stronger reduction in savings compared to the capital income subsidy scheme. Therefore, the superiority of lump-sum transfer we found in Figure 23a is due to a greater decrease in savings. However, this advantage of a greater savings reduction diminishes if we assign a lower pollution rate ξ and a higher interest rate R_{t+1}, as has been demonstrated in Figure 24. Lump-sum transfer scheme could be inferior to a capital income subsidy scheme if savings reduction yields unfavorable welfare impact. We have shown this case in Figure 11 and 12 for the calibrations with a low pollution rate ξ and a high interest rate R_{t+1}, respectively.

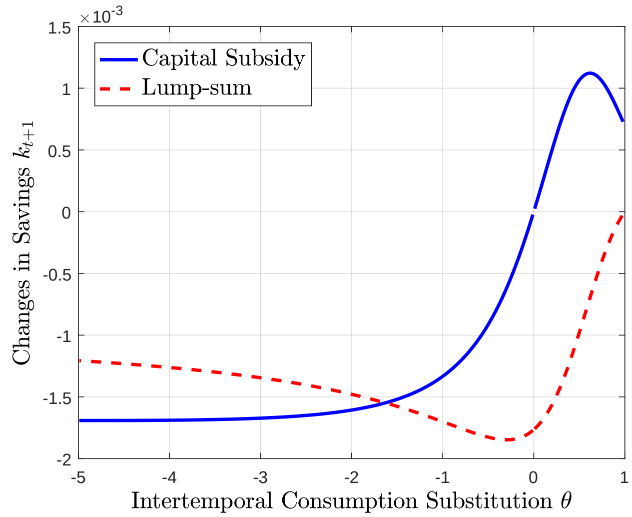


Figure 25: Intertemporal Substitutability Simulations for both Schemes with $\rho = 0$, $\xi = 1$, $k_0 = 1$, $A = 1.5$, $\tau^o_{t+1} = 0.01$, and $\mathcal{T}_{t+1} = 0.001$

These findings with varying degree of intertemporal substitution parameter θ is consistent with the findings with the Cobb-Douglas case ($\theta = 0$). The comparison of the $\mathcal{P}$-set between the lump-sum transfer scheme and the capital income subsidy scheme is largely determined by the capital accumulation externality and the interest rate. Intuitively, to determine which scheme has a higher Pareto-improving set of contracts, we need to assess their impacts on savings. The scheme that yields a greater reduction in savings has more Pareto-improving feasibility if a decrease in savings is beneficial for the households' welfare—high capital accumulation externality and low interest rate. Conversely, a reduction of savings may have an adverse impact on the households' welfare. If so, the scheme with a higher savings reduction would not be superior (or even become inferior if even a small decrease in savings greatly reduces the households' welfare).

5 Conclusions

The gap between the costs and benefits of climate mitigation efforts raises the difficulties in implementing climate policy if each generation is selfish. Nevertheless, existing studies argue that it is possible for each generation to mutually gain from climate mitigation efforts provided that the policy is combined with intergenerational transfers. Such a policy that benefits all generations is called a Pareto-improving policy. This thesis employs a two-period overlapping generations (OLG) model to test the existence of Pareto-improving social contracts. In this model, the households in each generation live for two periods: they work in one period and then retire in the next period. The social contracts consist of pollution mitigation performed by the *present-young generation* and a pay-as-you-go (PAYG) pension transfer from the *future-young generation*. Although the future generation needs to pay the tranfers, they benefit from the mitigation carried out by the previous generation. Signing the social contracts by these two successive generations may improve the welfare of both generations.

We have tested the existence of Pareto-improving social contracts under different schemes: the capital income subsidy scheme and the lump-sum transfer scheme. We also have extended the OLG model specification by Dao et al. (2017) with a different levels of consumption substitutability between the young-age and the old-age of consumption. We have shown that the Pareto-improving feasibility of initiating a social contract depends on whether reducing savings is beneficial for the households. A decrease in savings is more favorable if the externality from capital accumulation is high, and if the interest rate is low (compared to the population growth rate). Intuitively, reducing savings (which implies lower capital investments) also reduces the capital externality, and therefore it is more beneficial if the externality is high. If the economy has a low interest rate, a decrease in savings will increase the interest rate, so the households will receive a higher return from the crowding-out effect.

We have found that a lower degree of intertemporal consumption substitutability raises the Pareto-improving contract feasibility if reducing savings is advantageous. A lower degree of intertemporal substitution implies a greater degree of consumption smoothing by households. Signing a social contract changes the households' intertemporal consumption, that is, decreasing their young-age consumption budget while increasing their old-age consumption budget. Households with a lower degree of consumption substitutability will reduce their savings greatly to prevent such changes in their intertemporal consumption. Therefore, the greater reduction in savings increases the feasibility of Pareto-improving social contracts if the reducing savings is beneficial.

The arguments regarding the benefits of savings reduction are also related to the comparison of the Pareto-improving feasibility between the transfer schemes. We have demonstrated that a lump-sum transfer scheme is superior if it yields a stronger savings reduction and if discouraging savings is beneficial. The lump-sum transfer scheme discourages savings more than the capital income subsidy scheme when the intertemporal consumption substitutability is high. Households who are willing to substitute their intertemporal consumption tend to increase their savings to gain more from a capital subsidy, whereas this is not the case for a lump-sum transfer.

This thesis has several limitations which could be improved for future research. First, we have assumed a constant population. Population growth rates may affect the feasibility of Pareto-improving social contracts since it affects the benefits of discouraging savings. Second, we have not addressed the commitment concerns. Because the intergenerational social contract typically involves an *unborn* generation, this generation may refuse to pay the transfers even if the previous generation has fulfilled the mitigation efforts. Third, the comparison between the two schemes that we have conducted relies on the highest mitigation share of the Pareto-improving set of social contracts. A more proper welfare analysis between the two schemes should be employed instead. Nevertheless, this thesis contributes to the literature related to intergenerational transfers. We have shown the importance of evaluating the advantage of discouraging savings in the initial economic conditions, which influence the feasibility of Pareto-improving social contracts.

Acknowledgement

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Appendix

A Capital Income Subsidy

A.1 Households Utility Maximization

$$\begin{aligned}
&\max_{c_t^y, k_{t+1}, c_{t+1}^o} \quad \mathcal{V}_t = \frac{(c_t^y)^\theta}{\theta} + \beta\frac{(c_{t+1}^o)^\theta}{\theta} \\
&\text{subject to} \\
&c_t^y + k_{t+1} = I_t(1-m_t) \\
&c_{t+1}^o = R_{t+1}k_{t+1}(1+\tau_{t+1}^o)
\end{aligned} \tag{A.1}$$

Here, we can write the Lagrangian function:

$$\mathcal{L} = \frac{(c_t^y)^\theta}{\theta} + \beta\frac{(c_{t+1}^o)^\theta}{\theta} + \lambda_1(c_t^y + k_{t+1} - I_t(1-m_t)) + \lambda_2(c_{t+1}^o - R_{t+1}k_{t+1}(1+\tau_{t+1}^o))$$

The first order conditions:

$$\begin{aligned}
&\frac{\partial\mathcal{L}}{c_t^y} = (c_t^y)^{\theta-1} + \lambda_1 = 0 \\
&\frac{\partial\mathcal{L}}{c_{t+1}^o} = \beta(c_{t+1}^o)^{\theta-1} + \lambda_2 = 0 \\
&\frac{\partial\mathcal{L}}{k_{t+1}} = \lambda_1 - \lambda_2 R_{t+1}(1+\tau_{t+1}^o) = 0 \\
&\frac{\partial\mathcal{L}}{\lambda_1} = c_t^y + k_{t+1} - I_t(1-m_t) = 0 \\
&\frac{\partial\mathcal{L}}{\lambda_2} = c_{t+1}^o - R_{t+1}k_{t+1}(1+\tau_{t+1}^o) = 0
\end{aligned} \tag{A.2}$$

The division between the first and the second partial derivatives of (A.2) result in the ratio $\frac{\lambda_1}{\lambda_2} = \frac{1}{\beta}\left(\frac{c_t^y}{c_{t+1}^o}\right)^{\theta-1}$. The third partial derivative results in the ratio $\frac{\lambda_1}{\lambda_2} = R_{t+1}(1+\tau_{t+1}^o)$. Equating them will yield the equations:

$$\frac{1}{\beta}\left(\frac{c_t^y}{c_{t+1}^o}\right)^{\theta-1} = R_{t+1}(1+\tau_{t+1}^o) \tag{A.3}$$

from the fourth and the fifth partial derivatives in (A.2), we have the result:

$$\frac{I_t(1-m_t) - k_{t+1}}{R_{t+1}k_{t+1}(1+\tau_{t+1}^o)} = [\beta R_{t+1}(1+\tau_{t+1}^o)]^{\frac{1}{\theta-1}} \tag{A.4}$$

which could be solved for k_{t+1}:

$$I_t(1-m_t) - k_{t+1} = k_{t+1}\beta^{\frac{1}{\theta-1}}[R_{t+1}(1+\tau^o_{t+1})]^{\frac{\theta}{\theta-1}}$$
$$k_{t+1} = I_t(1-m_t)\left[\frac{1}{1+\beta^{\frac{1}{\theta-1}}[R_{t+1}(1+\tau^o_{t+1})]^{\frac{\theta}{\theta-1}}}\right] \tag{A.5}$$

Which refer to equation (3.4) for the optimal saving decision. substituting k_{t+1} into the budget constraints, we have the optimal consumption decision:

$$c^y_t = I_t(1-m_t)\left[1-\frac{1}{1+\beta^{\frac{1}{\theta-1}}[R_{t+1}(1+\tau^o_{t+1})]^{\frac{\theta}{\theta-1}}}\right] \tag{A.6}$$

and

$$c^o_{t+1} = I_t(1-m_t)R_{t+1}(1+\tau^o_{t+1})\left[\frac{1}{1+\beta^{\frac{1}{\theta-1}}[R_{t+1}(1+\tau^o_{t+1})]^{\frac{\theta}{\theta-1}}}\right] \tag{A.7}$$

which refer to (3.5) and (3.6) respectively.

A.2 Profit Maximization

The maximization problem is as follows:

$$\underset{K_t,L_t}{max} \;\; \Pi = p_tY_t - R_tK_t - w_tL_t \tag{A.8}$$

where p_t is price at period t, we also normalize it to one. Y_t is the CES production function.

$$Y_t = z(E_{t-1})[\alpha(K_t)^\rho + (1-\alpha)(L_t)^\rho]^{\frac{1}{\rho}} \tag{A.9}$$

R_t is the rate of return of capital, and w_t is the wage rate at time t.

the first order conditions:

$$\frac{\partial \Pi}{\partial K_t} = \alpha z(E_{t-1})(K_t)^{\rho-1}[\alpha(K_t)^\rho + (1-\alpha)(L_t)^\rho]^{\frac{1-\rho}{\rho}} - R_t = 0$$

so

$$R_t = \alpha z(E_{t-1})\left[\alpha + (1-\alpha)\left(\frac{L_t}{K_t}\right)^\rho\right]^{\frac{1-\rho}{\rho}}$$

We could write it in intensive form:

$$R_t = \alpha z(E_{t-1})\left[\alpha + (1-\alpha)(k_t)^{-\rho}\right]^{\frac{1-\rho}{\rho}} \tag{A.10}$$

for the labor demand, we have:

$$\frac{\partial \Pi}{\partial L_t} = (1-\alpha)z(E_{t-1})(L_t)^{\rho-1}[\alpha(K)_t^\rho + (1-\alpha)(L_t)^\rho]^{\frac{1-\rho}{\rho}} - w_t = 0$$

or, when written in intensive form, we have:

$$w_t = (1-\alpha)z(E_{t-1})\left[\alpha(k_t)^\rho + (1-\alpha)\right]^{\frac{1-\rho}{\rho}} \tag{A.11}$$

It is also useful to determine the ratio of R_t and w_t to eliminate most of the complexities in the model:

$$\frac{\alpha}{1-\alpha}\left(\frac{K_t}{L_t}\right)^{\rho-1} = \frac{R_t}{w_t}$$

or, when written in intensive form:

$$(k_t)^{\rho-1} = \frac{1-\alpha}{\alpha}\frac{R_t}{w_t} \tag{A.12}$$

A.3 Comparative-Static Analysis

We derive the partial derivatives of k_{t+1} w.r.t m_t:

$$\begin{aligned}
k_{t+1} &= s(R_{t+1}, \tau^o_{t+1})I_t(1-m_t) \\
\frac{\partial k_{t+1}}{\partial m_t} &= \frac{\partial s(R_{t+1}, \tau^o_{t+1})}{\partial m_t} I_t(1-m_t) - I_t s(R_{t+1}, \tau^o_{t+1}) \\
\frac{\partial k_{t+1}}{\partial m_t} &= \frac{\partial s(R_{t+1}, \tau^o_{t+1})}{\partial R_{t+1}} \frac{\partial R_{t+1}}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial m_t} I_t(1-m_t) - I_t s(R_{t+1}, \tau^o_{t+1}) \\
\frac{\partial k_{t+1}}{\partial m_t} &= -\frac{I_t s(R_{t+1}, \tau^o_{t+1})}{\left(1 - \frac{\partial s(R_{t+1}, \tau^o_{t+1})}{\partial R_{t+1}} \frac{\partial R_{t+1}}{\partial k_{t+1}} I_t(1-m_t)\right)}.
\end{aligned} \tag{A.13}$$

The partial derivative of savings rate w.r.t. R_{t+1} is

$$\frac{\partial s(R_{t+1}, \tau^o_{t+1})}{\partial R_{t+1}} = \frac{\theta}{\theta-1} \frac{[s(R_{t+1}, \tau^o_{t+1})^2 - s(R_{t+1}, \tau^o_{t+1})]}{R_{t+1}},$$

and substituting $I_t(1-m_t) = \frac{k_{t+1}}{s(R_{t+1},\tau^o_{t+1})}$, we have the following equations:

$$\begin{aligned}
\frac{\partial k_{t+1}}{\partial m_t} &= -\frac{I_t s(R_{t+1},\tau^o_{t+1})}{\left(1-\frac{\theta}{\theta-1}\frac{\partial R_{t+1}}{\partial k_{t+1}}\frac{[s(R_{t+1},\tau^o_{t+1})^2 - s(R_{t+1},\tau^o_{t+1})]}{R_{t+1}}\frac{k_{t+1}}{s(R_{t+1},\tau^o_{t+1})}\right)} \\
\frac{\partial k_{t+1}}{\partial m_t} &= -\frac{I_t s(R_{t+1},\tau^o_{t+1})}{\left(1-\frac{\theta}{\theta-1}\frac{\partial R_{t+1}}{\partial k_{t+1}}\frac{k_{t+1}}{R_{t+1}}(s(R_{t+1},\tau^o_{t+1})-1)\right)} \\
\frac{\partial k_{t+1}}{\partial m_t} &= -\frac{I_t s(R_{t+1},\tau^o_{t+1})}{\left(1+\frac{\theta}{\theta-1}\varepsilon_{R,k}\big(1-s(R_{t+1},\tau^o_{t+1})\big)\right)}.
\end{aligned} \tag{A.14}$$

The last expression corresponds to equation (3.24). For the effects of m_t on consumption c^y_t :

$$\begin{aligned}
c^y_t &= I_t(1-m_t) - k_{t+1} \\
\frac{\partial c^y_t}{\partial m_t} &= -I_t - \frac{\partial k_{t+1}}{\partial m_t}
\end{aligned} \tag{A.15}$$

The last expression refers to (3.25). For the effect on c^o_{t+1} :

$$\begin{aligned}
c^o_{t+1} &= R_{t+1}k_{t+1}(1+\tau^o_{t+1}) \\
\frac{\partial c^o_{t+1}}{\partial m_t} &= \left(\frac{\partial R_{t+1}}{\partial m_t}k_{t+1} + R_{t+1}\frac{\partial k_{t+1}}{\partial m_t}\right)(1+\tau^o_{t+1}) \\
\frac{\partial c^o_{t+1}}{\partial m_t} &= R_{t+1}(1+\tau^o_{t+1})\left(\frac{\partial R_{t+1}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial m_t}\frac{k_{t+1}}{R_{t+1}} + \frac{\partial k_{t+1}}{\partial m_t}\right) \\
\frac{\partial c^o_{t+1}}{\partial m_t} &= R_{t+1}(1+\tau^o_{t+1})\frac{\partial k_{t+1}}{\partial m_t}\left(\frac{\partial R_{t+1}}{\partial k_{t+1}}\frac{k_{t+1}}{R_{t+1}} + 1\right) \\
\frac{\partial c^o_{t+1}}{\partial m_t} &= \frac{\partial k_{t+1}}{\partial m_t}(\varepsilon_{R,k}+1)R_{t+1}(1+\tau^o_{t+1})
\end{aligned} \tag{A.16}$$

The last expression corresponds to (3.26). Next, we derive the partial derivative of k_{t+1} w.r.t. τ^o_{t+1}:

$$\begin{aligned}
k_{t+1} &= s(R_{t+1},\tau^o_{t+1})I_t(1-m_t) \\
\frac{\partial k_{t+1}}{\partial \tau^o_{t+1}} &= \frac{\partial s(R_{t+1},\tau^o_{t+1})}{\partial \tau^o_{t+1}}I_t(1-m_t)
\end{aligned} \tag{A.17}$$

The partial derivative of savings rate w.r.t. τ^o_{t+1}:

$$\begin{aligned}
s(R_{t+1},\tau^o_{t+1}) &= \left(\frac{1}{1+[\beta^{\frac{1}{\theta}}R_{t+1}(1+\tau^o_{t+1})]^{\frac{\theta}{\theta-1}}}\right)\\
\frac{\partial s(R_{t+1},\tau^o_{t+1})}{\partial \tau^o_{t+1}} &= -\left(\frac{1}{1+[\beta^{\frac{1}{\theta}}R_{t+1}(1+\tau^o_{t+1})]^{\frac{\theta}{\theta-1}}}\right)^2 \frac{\theta}{\theta-1}\frac{[\beta^{\frac{1}{\theta}}R_{t+1}(1+\tau^o_{t+1})]^{\frac{\theta}{\theta-1}}}{(1+\tau^o_{t+1})}\\
\frac{\partial s(R_{t+1},\tau^o_{t+1})}{\partial \tau^o_{t+1}} &= -\frac{\theta}{\theta-1}s(R_{t+1},\tau^o_{t+1})^2\left(\frac{1-s(R_{t+1},\tau^o_{t+1})}{s(R_{t+1},\tau^o_{t+1})}\right)\frac{1}{(1+\tau^o_{t+1})}\\
\frac{\partial s(R_{t+1},\tau^o_{t+1})}{\partial \tau^o_{t+1}} &= -\frac{\theta}{\theta-1}s(R_{t+1},\tau^o_{t+1})\left(\frac{1-s(R_{t+1},\tau^o_{t+1})}{(1+\tau^o_{t+1})}\right)
\end{aligned} \tag{A.18}$$

Substituting to the (A.17), we have the following expressions:

$$\begin{aligned}
\frac{\partial k_{t+1}}{\partial \tau^o_{t+1}} &= -\frac{\theta}{\theta-1}s(R_{t+1},\tau^o_{t+1})\left(\frac{1-s(R_{t+1},\tau^o_{t+1})}{(1+\tau^o_{t+1})}\right)I_t(1-m_t)\\
\frac{\partial k_{t+1}}{\partial \tau^o_{t+1}} &= -\frac{\theta}{\theta-1}s(R_{t+1},\tau^o_{t+1})\left(\frac{1-s(R_{t+1},\tau^o_{t+1})}{(1+\tau^o_{t+1})}\right)\frac{k_{t+1}}{s(R_{t+1},\tau^o_{t+1})}\\
\frac{\partial k_{t+1}}{\partial \tau^o_{t+1}} &= -\frac{\theta}{\theta-1}k_{t+1}\left(\frac{1-s(R_{t+1},\tau^o_{t+1})}{(1+\tau^o_{t+1})}\right)
\end{aligned} \tag{A.19}$$

The last expression refers to (3.27). Next,we derive the partial derivatives for consumption:

$$\begin{aligned}
c^y_t &= I_t(1-m_t)-k_{t+1}\\
\frac{\partial c^y_t}{\partial \tau^o_{t+1}} &= -\frac{\partial k_{t+1}}{\partial \tau^o_{t+1}}.
\end{aligned} \tag{A.20}$$

These expressions is the derivations of (3.28). and the old-age consummption:

$$\begin{aligned}
c^o_{t+1} &= R_{t+1}k_{t+1}(1+\tau^o_{t+1})\\
\frac{\partial c^o_{t+1}}{\partial \tau^o_{t+1}} &= \frac{\partial R_{t+1}}{\partial \tau^o_{t+1}}k_{t+1}(1+\tau^o_{t+1})+R_{t+1}\frac{\partial k_{t+1}}{\partial \tau^o_{t+1}}(1+\tau^o_{t+1})+R_{t+1}k_{t+1}\\
\frac{\partial c^o_{t+1}}{\partial \tau^o_{t+1}} &= \frac{\partial k_{t+1}}{\partial \tau^o_{t+1}}(1+\tau^o_{t+1})\left(\frac{\partial R_{t+1}}{\partial k_{t+1}}k_{t+1}+R_{t+1}\right)+R_{t+1}k_{t+1}
\end{aligned} \tag{A.21}$$

Substituting (3.27) regarding $\frac{\partial k_{t+1}}{\partial \tau^o_{t+1}}$, we have:

$$
\begin{aligned}
\frac{\partial c^o_{t+1}}{\partial \tau^o_{t+1}} &= -\frac{\theta}{\theta-1} k_{t+1} \left(\frac{1 - s(R_{t+1}, \tau^o_{t+1})}{(1+\tau^o_{t+1})} \right) (1+\tau^o_{t+1}) \left(\frac{\partial R_{t+1}}{\partial k_{t+1}} k_{t+1} + R_{t+1} \right) + R_{t+1} k_{t+1} \\
\frac{\partial c^o_{t+1}}{\partial \tau^o_{t+1}} &= -\frac{\theta}{\theta-1} R_{t+1} k_{t+1} \left(1 - s(R_{t+1}, \tau^o_{t+1})\right) (\varepsilon_{R,k} + 1) + R_{t+1} k_{t+1} \\
\frac{\partial c^o_{t+1}}{\partial \tau^o_{t+1}} &= R_{t+1} k_{t+1} \left(1 - \frac{\theta}{\theta-1} \left(1 - s(R_{t+1}, \tau^o_{t+1})\right) (\varepsilon_{R,k} + 1) \right)
\end{aligned}
\tag{A.22}
$$

the last expression is the derivations for (3.29).

A.4 Cobb-Douglas Case

Households

We apply a limiting argument of $\lim_{\theta \to 0}$ to the utility function in (A.1):

$$
\lim_{\theta \to 0} \mathcal{V}_t = [(c^y_t)^\theta + \beta (c^o_{t+1})^\theta]^{\frac{1}{\theta}} \tag{A.23}
$$

Because assigning $\theta = 0$ directly to the equation yield to undefined term (power of ∞), we employ *L'Hôpital's rule* to find the limit. To apply *L'Hôpital's rule*, we need to transform (A.23) into the following expression:

$$
\lim_{\theta \to 0} \ln \left(\frac{\mathcal{V}_t}{(1+\beta)^{\frac{1}{\theta}}} \right) = \frac{\ln \frac{(c^y_t)^\theta + \beta (c^o_{t+1})^\theta}{1+\beta}}{\theta} \tag{A.24}
$$

From this equation, substituting $\theta = 0$ will yield a 0/0 result. Taking the derivative of both the denominator and the numerator with respect to θ:

$$
\begin{aligned}
\lim_{\theta \to 0} \ln \left(\frac{\mathcal{V}_t}{(1+\beta)^{\frac{1}{\theta}}} \right) &= \frac{1+\beta}{(c^y_t)^\theta + \beta (c^o_{t+1})^\theta} [(c^y_t)^\theta \ln(c^y_t) + \beta (c^o_{t+1})^\theta \ln(c^o_{t+1})] \\
&= \ln(c^y_t) + \beta \ln(c^o_{t+1}) \quad \text{by substituting } \theta = 0
\end{aligned}
\tag{A.25}
$$

The last expression of (A.25) is the logarithmic utility function employed by Dao et al. (2017). The optimal consumption and savings choices will coincide with the logarithmic utility function in Dao et al. (2017). Their Cobb-Douglas utility of $(c^y_t)(c^o_{t+1})^\beta$ or equivalent to $\ln(c^y_t) + \beta \ln(c^o_{t+1})$ implies that the θ in the CES utility function (A.1) equal to 0. Applying $\theta = 0$ in equation (A.5), (A.6), and (A.7), we have the same result as in Dao et al. (2017).

$$
\begin{aligned}
k_{t+1} &= I_t(1-m_t)\left(\frac{\beta}{1+\beta}\right) \\
c_t^y &= I_t(1-m_t)\left[1-\left(\frac{\beta}{1+\beta}\right)\right] \\
c_{t+1}^o &= I_t(1-m_t)R_{t+1}(1+\tau_{t+1}^o)\left[\frac{\beta}{1+\beta}\right]
\end{aligned}
\tag{A.26}
$$

Firms and Equilibrium

The case of Cobb-Douglas production function would have $\rho = 0$. We need to use limiting argument for (A.10) and (A.11). We transform (A.10) into the following expression to apply *L'Hôpital's rule*

$$
\begin{aligned}
\lim_{\rho\to 0} R_t &= \exp\left(\lim_{\rho\to 0} \ln R_t\right) \\
&= \exp\left(\lim_{\rho\to 0}\left(\ln\alpha + \ln z(E_{t-1}) + \frac{1-\rho}{\rho}\ln(\alpha + (1-\alpha)(k_t)^{-\rho})\right)\right)
\end{aligned}
\tag{A.27}
$$

If we substitute $\rho = 0$, the last term would result in 0/0 expression. We could therefore apply *L'Hôpital's rule* by taking the derivatives of the numerator and denominator with respect to ρ.

$$
\begin{aligned}
\lim_{\rho\to 0} R_t &= \exp\left(\lim_{\rho\to 0}\left(\ln\alpha + \ln z(E_{t-1}) + \frac{(1-\rho)(\alpha-1)(k_t)^{-\rho}\ln k_t}{\alpha+(1-\alpha)(k_t)^{-\rho}} - \ln(\alpha+(1-\alpha)(k_t)^{-\rho})\right)\right) \\
&= \exp\left(\ln\alpha + \ln z(E_{t-1}) + \ln k_t^{(\alpha-1)}\right) \quad \text{by substituting } \rho = 0 \\
&= \alpha z(E_{t-1})k_t^{(\alpha-1)}
\end{aligned}
\tag{A.28}
$$

Using similar procedure for w_t in (A.11), we could get the following derivations:

$$
\begin{aligned}
\lim_{\rho\to 0} w_t &= \exp\left(\lim_{\rho\to 0}\left(\ln(1-\alpha) + \ln z(E_{t-1}) + \frac{(1-\rho)\alpha(k_t)^{\rho}\ln k_t}{\alpha(k_t)^{\rho}+(1-\alpha)} - \ln(\alpha(k_t)^{\rho}+(1-\alpha))\right)\right) \\
&= \exp\left(\ln(1-\alpha) + \ln z(E_{t-1}) + \ln(k_t)^{\alpha}\right) \\
&= (1-\alpha)z(E_{t-1})(k_t)^{\alpha}
\end{aligned}
\tag{A.29}
$$

Both of the last expressions in (A.28) and (A.29) is the same as in the Cobb-Douglas Production function in Dao et al. (2017). We summarise the system of equations in Table 4.

Table 4: System of Equations for Capital Income Subsidy Scheme–Cobb-Douglas Case

$$c_t^y = I_t(1-m_t)\left[1-\left(\frac{\beta}{1+\beta}\right)\right] \tag{A.30}$$

$$c_{t+1}^o = I_t(1-m_t)R_{t+1}(1+\tau_{t+1}^o)\left[\frac{\beta}{1+\beta}\right] \tag{A.31}$$

$$k_{t+1} = I_t(1-m_t)\left(\frac{\beta}{1+\beta}\right) \tag{A.32}$$

$$I_t = (1-\alpha)z(E_{t-1})(k_t)^\alpha\left(1-\frac{\alpha}{1-\alpha}(\tau_t^o)\right) \tag{A.33}$$

$$R_{t+1} = \alpha z(E_t)k_{t+1}^{(\alpha-1)} \tag{A.34}$$

$$E_t = (1-\delta)E_{t-1} + \xi k_t - \gamma m_{t-1}I_{t-1} \tag{A.35}$$

$$z(E_t) = Ae^{-|E_t|} \tag{A.36}$$

Pareto-Improving Contracts with Cobb-Douglas Functions

To derive the sets Pareto-improving social contracts with capital income subsidy scheme analytically, we first derive the utility gains for both $\mathcal{G}_t$ and $\mathcal{G}_{t+1}$.

For $\mathcal{G}_t$: We substitute the optimal choices in (A.26) into the logarithmic utility function in (A.25):

$$\mathcal{V}_t^c = \ln\left(\frac{I_t(1-m_t)}{1+\beta}\right) + \beta\ln\left(I_t(1-m_t)R_{t+1}(1+\tau_{t+1}^o)\left[\frac{\beta}{1+\beta}\right]\right) \tag{A.37}$$

where $I_t = w_t(1-)\tau_t^y$ and $\tau_t^y = \frac{\alpha}{1-\alpha}\tau_t^o$ from equation (3.23) if $\rho = 0$. We then substitute R_t from (A.28), so we have the following expression:

$$\mathcal{V}_t^c = \ln\left(\frac{I_t(1-m_t)}{1+\beta}\right) + \beta\ln\left(I_t(1-m_t)\alpha z(E_t)(k_{t+1})^{\alpha-1}(1+\tau_{t+1}^o)\left[\frac{\beta}{1+\beta}\right]\right) \tag{A.38}$$

Now, we can compare the lifetime utility for $\mathcal{G}_t$ without social contract by imposing $m_t =$

$\tau^o_{t+1} = 0$, and denoting endogenous variables under no social contract by tilde.

$$\mathcal{V}^0_t = \ln\left(\frac{\tilde{I}_t}{1+\beta}\right) + \beta \ln\left(\tilde{I}_t \alpha z(\tilde{E}_t)(\tilde{k}_{t+1})^{\alpha-1}\left[\frac{\beta}{1+\beta}\right]\right) \tag{A.39}$$

The symbols $\tilde{I}_t$, $z(\tilde{E}_t)$, and $\tilde{k}_t$ represents the net income, total productivity factor, and capital per capita under no social contract at for generation t, respectively. The difference of lifetime utility between with and without social contracts:

$$\Delta\mathcal{V}^c_t = \ln\left(\frac{I_t}{\tilde{I}_t}(1-m_t)\right) + \beta \ln\left(\frac{I_t}{\tilde{I}_t}(1-m_t)\left(\frac{z(E_t)}{z(\tilde{E}_t)}\right)\left(\frac{k_{t+1}}{\tilde{k}_{t+1}}\right)^{\alpha-1}(1+\tau^o_{t+1})\right) \tag{A.40}$$

Because we consider only the social contract (m_t, τ^o_{t+1}), the the pollution stock E_t (see equation (3.21)) is the same, with or without social contract. It involves only the previous period mitigation share and net income where $M_t = m_{t-1}I_{t-1}$. Thus, we could assume $z(\tilde{E}_t) = z(E_t)$ and $z(\tilde{E}_{t-1}) = z(E_{t-1})$.Furthermore, the mitigation share m_t affect only k_{t+1} (see equation (A.26)), not the saving decision in the previous period k_t, we could assume $\tilde{k}_t = k_t$. It also implies that $\tilde{\tau}^o_t = \tau^o_t$. From (A.29), $I_t = w_t(1-\tau^y_t)$, and $\tau^y_t = \frac{\alpha}{1-\alpha}\tau^o_t$, we derive that:

$$\frac{I_t}{\tilde{I}_t} = \frac{(1-\alpha)z(E_{t-1})(k_t)^{\alpha}\left(1-\frac{\alpha}{1-\alpha}\tau^o_t\right)}{(1-\alpha)z(\tilde{E}_{t-1})(\tilde{k}_t)^{\alpha}\left(1-\frac{\alpha}{1-\alpha}\tilde{\tau}^o_t\right)} = 1$$

Based on (A.26), we could also derive that $\frac{k_{t+1}}{\tilde{k}_{t+1}} = (1-m_t)$. We conclude with *utility gains from social contract for* $\mathcal{G}_t$:

$$\Delta\mathcal{V}^c_t = (1+\alpha\beta)\ln(1-m_t) + \beta\ln(1+\tau^o_{t+1}). \tag{A.41}$$

For $\mathcal{G}_{t+1}$: We shift equation (A.38) and (A.39) by one period:

$$\mathcal{V}^c_{t+1} = \ln\left(\frac{I_{t+1}(1-m^e_{t+1})}{1+\beta}\right) + \beta\ln\left(I_t(1-m^e_{t+1})\alpha z(E_{t+1})(k_{t+2})^{\alpha-1}(1+\tau^{o,e}_{t+2})\left[\frac{\beta}{1+\beta}\right]\right) \tag{A.42}$$

where $(m^e_{t+1}, \tau^{o,e}_{t+2})$ is the expected social contract after social contract (m_t, τ^o_{t+1}). For the lifetime utility without social contract, we again impose $m_t = \tau^o_{t+1} = 0$. We also derive the lifetime utility for generation $t+1$ under no social contract $(m_t, \tau^o_{t+1}) = (0,0)$.

$$\mathcal{V}^0_{t+1} = \ln\left(\frac{\tilde{I}_{t+1}(1-\tilde{m}^e_{t+1})}{1+\beta}\right) + \beta\ln\left(\tilde{I}_t(1-\tilde{m}^e_{t+1})\alpha z(\tilde{E}_{t+1})(\tilde{k}_{t+2})^{\alpha-1}(1+\tilde{\tau}^{o,e}_{t+2})\left[\frac{\beta}{1+\beta}\right]\right) \tag{A.43}$$

With the argument that the preceding contract (m_{t-1}, τ^o_t) and succeeding contract $(m^e_{t+1}, \tau^{o,e}_{t+2})$ do not affect the utility gains, it implies that $\tilde{\tau}^{o,e}_{t+2} = \tau^{o,e}_{t+2}$ and $\tilde{m}^e_{t+1} = m^e_{t+1}$. The utility gains for generation $t+1$:

$$\Delta\mathcal{V}^c_{t+1} = (1+\beta)\ln\left(\frac{I_{t+1}}{\tilde{I}_{t+1}}\right) + \beta\ln\left(\frac{z(\tilde{E}_{t+1})}{z(E_{t+1})}\right) + \beta(\alpha-1)\ln\left(\frac{k_{t+2}}{\tilde{k}_{t+2}}\right) \tag{A.44}$$

From (A.26) and (A.29), we could derive that $k_{t+2} = I_{t+1}(1-m_{t+1})\frac{\beta}{1+\beta}$ and $I_{t+1} = (1-\alpha)z(E_t)(k_{t+1})^\alpha\left(1 - \frac{\alpha}{1-\alpha}\tau^o_{t+1}\right)$. So the ratio

$$\frac{k_{t+2}}{\tilde{k}_{t+2}} = \frac{I_{t+1}}{\tilde{I}_{t+1}} = \left(\frac{k_{t+1}}{\tilde{k}_{t+1}}\right)^\alpha\left(1 - \frac{\alpha}{1-\alpha}\tau^o_{t+1}\right)$$

We call the pollution stock equation (3.9) and total factor productivity (3.22) to find the ratio:

$$\frac{z(E_{t+1})}{z(\tilde{E}_{t+1})} = \frac{e^{-|(1-\delta)E_t + k_{t+1} - \gamma m_t I_t|}}{e^{-|(1-\delta)\tilde{E}_t + \tilde{k}_{t+1}|}} = e^{\tilde{k}_{t+1} - k_{t+1} + \gamma m_t I_t} = e^{\frac{\beta+\gamma+\beta\gamma}{1+\beta}m_t I_t}$$

we use again the expression $\frac{k_{t+1}}{\tilde{k}_{t+1}} = (1-m_t)$. Therefore, *utility gains from social contract (m_t, τ^o_{t+1}) for $\mathcal{G}_{t+1}$*:

$$\Delta\mathcal{V}^c_{t+1} = (\alpha\beta+1)\ln\left(\left(1 - \frac{\alpha}{1-\alpha}\tau^o_{t+1}\right)(1-m_t)^\alpha\right) + \frac{\beta(\beta+\gamma+\beta\gamma)}{1+\beta}m_t I_t. \tag{A.45}$$

Sets of Pareto-Improving Contracts : We use expressions (A.41) and (A.45), set them equal to zero. For $\mathcal{G}_t$:

$$\begin{aligned} \Delta\mathcal{V}^c_t &= (1+\alpha\beta)\ln(1-m_t) + \beta\ln(1+\tau^o_{t+1}) \geq 0 \\ \ln(1+\tau^o_{t+1})^\beta &\geq \ln(1-m_t)^{-(1+\alpha\beta)} \\ \tau^o_{t+1} &\geq \left(\frac{1}{1-m_t}\right)^{\frac{\alpha\beta+1}{\beta}} - 1 \equiv \Omega(m_t) \end{aligned} \tag{A.46}$$

For the $\mathcal{G}_{t+1}$:

$$
\begin{aligned}
\Delta\mathcal{V}^c_{t+1} = (\alpha\beta+1)\ln\left(\left(1-\frac{\alpha}{1-\alpha}\tau^o_{t+1}\right)(1-m_t)^\alpha\right) + \frac{\beta(\beta+\gamma+\beta\gamma)}{1+\beta}m_tI_t \geq 0 \\
\left(1-\frac{\alpha}{1-\alpha}\tau^o_{t+1}\right)(1-m_t)^\alpha \geq e^{-\frac{\beta(\beta+\gamma+\beta\gamma)}{(1+\beta)(\alpha\beta+1)}m_tI_t} \\
\tau^o_{t+1} \leq \frac{1-\alpha}{\alpha}\left[1-e^{-\frac{\beta(\beta+\gamma+\beta\gamma)}{(1+\beta)(\alpha\beta+1)}m_tI_t}(1-m_t)^{-\alpha}\right] \equiv \psi(m_t,I_t)
\end{aligned}
\tag{A.47}
$$

Income Threshold : The derivation for the income threshold requires the slope of $\psi(m_t, I_t)$ and $\Omega(m_t)$ to be equal at $\lim_{m_t\to 0^+}$:

$$
\lim_{m_t\to 0^+}\frac{\partial\psi(m_t,I_t)}{\partial m_t} = \lim_{m_t\to 0^+}\frac{\partial\Omega(m_t)}{\partial m_t} \tag{A.48}
$$

$$
\begin{aligned}
&\lim_{m_t\to 0^+}\frac{\partial\psi(m_t,I_t)}{\partial m_t} \\
&= \lim_{m_t\to 0^+}\frac{1-\alpha}{\alpha}\left(e^{-\frac{\beta(\beta+\gamma+\beta\gamma)}{(1+\beta)(\alpha\beta+1)}m_tI_t}\right)\left[\frac{\beta(\beta+\gamma+\beta\gamma)}{(1+\beta)(\alpha\beta+1)}I_t(1-m_t)^{-\alpha} - \alpha(1-m_t)^{-\alpha-1}\right] \\
&= \frac{1-\alpha}{\alpha}\left[\frac{\beta(\beta+\gamma+\beta\gamma)}{(1+\beta)(\alpha\beta+1)}I_t - \alpha\right]
\end{aligned}
\tag{A.49}
$$

$$
\begin{aligned}
\lim_{m_t\to 0^+}\frac{\partial\Omega(m_t)}{\partial m_t} &= \lim_{m_t\to 0^+}\frac{\alpha\beta+1}{\beta}\left(\frac{1}{1-m_t}\right)^{\frac{\alpha\beta+\beta+1}{\beta}} \\
&= \frac{\alpha\beta+1}{\beta}
\end{aligned}
\tag{A.50}
$$

Therefore, the income threshold:

$$
\begin{aligned}
\frac{1-\alpha}{\alpha}\left[\frac{\beta(\beta+\gamma+\beta\gamma)}{(1+\beta)(\alpha\beta+1)}I_t - \alpha\right] &= \frac{\alpha\beta+1}{\beta} \\
\frac{\beta(\beta+\gamma+\beta\gamma)}{(1+\beta)(\alpha\beta+1)}I_t &= \frac{\alpha(1+\beta)}{\beta(1-\alpha)} \\
\hat{I}_t &= \frac{\alpha(1+\alpha\beta)(1+\beta)^2}{(1-\alpha)(\beta+\gamma+\gamma\beta)\beta^2}
\end{aligned}
\tag{A.51}
$$

Capital per Capita Threshold : Because we assume k_t as the exogeneous variable instead of income, we derive the threshold of k_t from (A.51):

$$(1-\alpha)z(E_{t-1})(k_t)^{\alpha}\,(1-\tau_t^y) = \frac{\alpha(1+\alpha\beta)(1+\beta)^2}{(1-\alpha)(\beta+\gamma+\gamma\beta)\beta^2}$$
$$(k_t)^{\alpha} = \frac{1}{(1-\alpha)z(E_{t-1})\,(1-\tau_t^y)}\frac{\alpha(1+\alpha\beta)(1+\beta)^2}{(1-\alpha)(\beta+\gamma+\gamma\beta)\beta^2}$$
$$\hat{k}_t = \left(\frac{1}{(1-\alpha)z(E_{t-1})\,(1-\tau_t^y)}\frac{\alpha(1+\alpha\beta)(1+\beta)^2}{(1-\alpha)(\beta+\gamma+\gamma\beta)\beta^2}\right)^{\frac{1}{\alpha}} \tag{A.52}$$

B Lump sum Transfers

B.1 Households Utility Maximization

$$\begin{aligned}
&\underset{c_t^y,k_{t+1},c_{t+1}^o}{max} \quad \mathcal{V}_t = \frac{(c_t^y)^\theta}{\theta} + \beta\frac{(c_{t+1}^o)^\theta}{\theta}\\
&\text{subject to}\\
&c_t^y + k_{t+1} = w_t(1-m_t) - \mathcal{T}_t\\
&c_{t+1}^o = R_{t+1}k_{t+1} + \mathcal{T}_{t+1}
\end{aligned} \tag{B.1}$$

Here, we can write the Lagrangian function:

$$\mathcal{L} = \frac{(c_t^y)^\theta}{\theta} + \beta\frac{(c_{t+1}^o)^\theta}{\theta} + \lambda_1(c_t^y + k_{t+1} - w_t(1-m_t) + \mathcal{T}_t) + \lambda_2(c_{t+1}^o - R_{t+1}k_{t+1} - \mathcal{T}_{t+1})$$

The first order conditions:

$$\begin{aligned}
&\frac{\partial\mathcal{L}}{c_t^y} = (c_t^y)^{\theta-1} + \lambda_1 = 0\\
&\frac{\partial\mathcal{L}}{c_{t+1}^o} = \beta(c_{t+1}^o)^{\theta-1} + \lambda_2 = 0\\
&\frac{\partial\mathcal{L}}{k_{t+1}} = \lambda_1 - \lambda_2 R_{t+1} = 0\\
&\frac{\partial\mathcal{L}}{\lambda_1} = c_t^y + k_{t+1} - w_t(1-m_t) + \mathcal{T}_t = 0\\
&\frac{\partial\mathcal{L}}{\lambda_2} = c_{t+1}^o - R_{t+1}k_{t+1} - \mathcal{T}_{t+1} = 0
\end{aligned} \tag{B.2}$$

The ratio of the first and the second partial derivative, combining with the third partial derivative result in

$$\frac{1}{\beta}\left(\frac{c_t^y}{c_{t+1}^o}\right)^{\theta-1} = R_{t+1} \iff c_t^y = c_{t+1}^o(\beta R_{t+1})^{\frac{1}{\theta-1}}$$

Substituting to the fourth and the fifth partial derivative, we could solve for k_{t+1}.

$$\begin{aligned}
(R_{t+1}k_{t+1} + \mathcal{T}_{t+1})(\beta R_{t+1})^{\frac{1}{\theta-1}} + k_{t+1} &= w_t(1-m_t) - \mathcal{T}_t \\
\beta^{\frac{1}{\theta-1}}(R_{t+1})^{\frac{\theta}{\theta-1}}k_{t+1} + k_{t+1} &= w_t(1-m_t) - \mathcal{T}_t - \mathcal{T}_{t+1}(\beta R_{t+1})^{\frac{1}{\theta-1}} \\
k_{t+1} &= \frac{w_t(1-m_t) - \mathcal{T}_t - \mathcal{T}_{t+1}(\beta R_{t+1})^{\frac{1}{\theta-1}}}{1+(\beta R_{t+1})^{\frac{1}{\theta-1}}R_{t+1}}
\end{aligned} \tag{B.3}$$

The last expression is the derivations of (3.32). For the firm decisions, the capital and labor employed will be the same as in the capital income subsidy case. The return on capital:

$$R_{t+1} = \alpha z(E_t)\left[\alpha + (1-\alpha)(k_{t+1})^{-\rho}\right]^{\frac{1-\rho}{\rho}} \tag{B.4}$$

Substituting to the k_{t+1} under the lump sum case.

$$k_{t+1} = \frac{w_t(1-m_t) - \mathcal{T}_t - \mathcal{T}_{t+1}(\beta\alpha z(E_t)\left[\alpha + (1-\alpha)(k_{t+1})^{-\rho}\right]^{\frac{1-\rho}{\rho}})^{\frac{1}{\theta-1}}}{1+(\beta^{\frac{1}{\theta}}\alpha z(E_t)\left[\alpha + (1-\alpha)(k_{t+1})^{-\rho}\right]^{\frac{1-\rho}{\rho}})^{\frac{\theta}{\theta-1}}} \tag{B.5}$$

We solve for k_{t+1} using numerical computations.

$$\begin{aligned}
k_{t+1} + k_{t+1}(\beta^{\frac{1}{\theta}}\alpha z(E_t)\left[\alpha + (1-\alpha)(k_{t+1})^{-\rho}\right]^{\frac{1-\rho}{\rho}})^{\frac{\theta}{\theta-1}} - w_t(1-m_t) + \mathcal{T}_t + \\
\mathcal{T}_{t+1}(\beta\alpha z(E_t)\left[\alpha + (1-\alpha)(k_{t+1})^{-\rho}\right]^{\frac{1-\rho}{\rho}})^{\frac{1}{\theta-1}} = 0
\end{aligned} \tag{B.6}$$

B.2 Comparative-Static Analysis

To study the effects of m_t on k_{t+1}, we derive the following expressions:

$$\begin{aligned}
k_{t+1} + \beta^{\frac{1}{\theta-1}}(R_{t+1})^{\frac{\theta}{\theta-1}}k_{t+1} + \mathcal{T}_t + \mathcal{T}_{t+1}(\beta R_{t+1})^{\frac{1}{\theta-1}} &= w_t(1-m_t) \\
\frac{\partial k_{t+1}}{\partial m_t}\left(1 + \beta^{\frac{1}{\theta-1}}(R_{t+1})^{\frac{\theta}{\theta-1}}\left(1 + \frac{\theta}{\theta-1}\frac{k_{t+1}}{R_{t+1}}\frac{\partial R_{t+1}}{\partial k_{t+1}} + \frac{1}{\theta-1}\mathcal{T}_{t+1}\frac{1}{(R_{t+1})^2}\frac{\partial R_{t+1}}{\partial k_{t+1}}\right)\right) &= -w_t \\
\frac{\partial k_{t+1}}{\partial m_t}\left(1 + \beta^{\frac{1}{\theta-1}}(R_{t+1})^{\frac{\theta}{\theta-1}}\left(1 + \frac{\theta}{\theta-1}\varepsilon_{R,k} + \frac{1}{\theta-1}\frac{\mathcal{T}_{t+1}}{k_{t+1}R_{t+1}}\varepsilon_{R,k}\right)\right) &= -w_t \\
\frac{\partial k_{t+1}}{\partial m_t}\left(1 + \beta^{\frac{1}{\theta-1}}(R_{t+1})^{\frac{\theta}{\theta-1}}\left(1 + \frac{\varepsilon_{R,k}}{\theta-1}\left(\theta + \frac{\mathcal{T}_{t+1}}{k_{t+1}R_{t+1}}\right)\right)\right) &= -w_t
\end{aligned} \tag{B.7}$$

Therefore,

$$\frac{\partial k_{t+1}}{\partial m_t} = -\frac{w_t}{\left(1+(\beta^{\frac{1}{\theta}}R_{t+1})^{\frac{\theta}{\theta-1}}\left(1+\frac{\varepsilon_{R,k}}{\theta-1}\left(\theta+\frac{\mathcal{T}_{t+1}}{k_{t+1}R_{t+1}}\right)\right)\right)} \tag{B.8}$$

This expression corresponds to (3.39).

Next, we derive the partial derivatives of c_t^y w.r.t. m_t:

$$\begin{aligned} c_t^y &= w_t(1-m_t) - \mathcal{T}_t - k_{t+1} \\ \frac{\partial c_t^y}{\partial m_t} &= -w_t - \frac{\partial k_{t+1}}{\partial m_t}. \end{aligned} \tag{B.9}$$

For partial derivative of c_{t+1}^o w.r.t. m_t:

$$\begin{aligned} c_{t+1}^o &= R_{t+1}k_{t+1} + \mathcal{T}_{t+1} \\ \frac{\partial c_{t+1}^o}{\partial m_t} &= \frac{\partial R_{t+1}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial m_t}k_{t+1} + R_{t+1}\frac{\partial k_{t+1}}{\partial m_t} \\ \frac{\partial c_{t+1}^o}{\partial m_t} &= \frac{\partial k_{t+1}}{\partial m_t}R_{t+1}\left(\varepsilon_{R,k}+1\right) \end{aligned} \tag{B.10}$$

For the partial derivatives w.r.t. $\mathcal{T}_{t+1}$, we derive the following expressions:

$$\begin{aligned} k_{t+1} + \beta^{\frac{1}{\theta-1}}(R_{t+1})^{\frac{\theta}{\theta-1}}k_{t+1} + \mathcal{T}_t + \mathcal{T}_{t+1}(\beta R_{t+1})^{\frac{1}{\theta-1}} &= w_t(1-m_t) \\ \frac{\partial k_{t+1}}{\mathcal{T}_{t+1}}\left(1 + \beta^{\frac{1}{\theta-1}}(R_{t+1})^{\frac{\theta}{\theta-1}} + \frac{\theta}{\theta-1}(\beta R_{t+1})^{\frac{1}{\theta-1}}\frac{\partial R_{t+1}}{\partial k_{t+1}}k_{t+1} + \frac{\mathcal{T}_{t+1}}{\theta-1}\frac{(\beta R_{t+1})^{\frac{1}{\theta-1}}}{R_{t+1}}\frac{\partial R_{t+1}}{\partial k_{t+1}}\right) &= -(\beta R_{t+1})^{\frac{1}{\theta-1}} \\ \frac{\partial k_{t+1}}{\mathcal{T}_{t+1}}\left(1 + \beta^{\frac{1}{\theta-1}}(R_{t+1})^{\frac{\theta}{\theta-1}}\left(1 + \frac{\theta}{\theta-1}\frac{k_{t+1}}{R_{t+1}}\frac{\partial R_{t+1}}{\partial k_{t+1}} + \frac{\mathcal{T}_{t+1}}{\theta-1}\frac{1}{R_{t+1}^2}\frac{\partial R_{t+1}}{\partial k_{t+1}}\right)\right) &= -(\beta R_{t+1})^{\frac{1}{\theta-1}} \end{aligned} \tag{B.11}$$

Therefore:

$$\frac{\partial k_{t+1}}{\partial \mathcal{T}_{t+1}} = -\frac{(\beta R_{t+1})^{\frac{1}{\theta-1}}}{1 + (\beta^{\frac{1}{\theta}}R_{t+1})^{\frac{\theta}{\theta-1}}\left(1 + \frac{\varepsilon_{R,k}}{\theta-1}\left(\theta + \frac{\mathcal{T}_{t+1}}{R_{t+1}k_{t+1}}\right)\right)} \tag{B.12}$$

For the consumptions:

$$\begin{aligned} c_t^y &= w_t(1-m_t) - \mathcal{T}_t - k_{t+1} \\ \frac{\partial c_t^y}{\partial \mathcal{T}_{t+1}} &= -\frac{\partial k_{t+1}}{\partial \mathcal{T}_{t+1}} \end{aligned} \tag{B.13}$$

$$\begin{aligned} c_{t+1}^o &= R_{t+1}k_{t+1} + \mathcal{T}_{t+1} \\ \frac{\partial c_{t+1}^o}{\partial \mathcal{T}_{t+1}} &= \frac{\partial R_{t+1}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial \mathcal{T}_{t+1}}k_{t+1} + R_{t+1}\frac{\partial k_{t+1}}{\partial \mathcal{T}_{t+1}} + 1 \\ \frac{\partial c_{t+1}^o}{\partial \mathcal{T}_{t+1}} &= \frac{\partial k_{t+1}}{\partial \mathcal{T}_{t+1}}R_{t+1}\left(\varepsilon_{R,k}+1\right) + 1 \end{aligned} \tag{B.14}$$

B.3 Cobb-Douglas Case

$$\begin{aligned}
&\max_{c_t^y, k_{t+1}, c_{t+1}^o} \quad \mathcal{V}_t = \ln c_t^y + \beta \ln c_{t+1}^o \\
&\text{subject to} \\
&c_t^y + k_{t+1} = w_t(1-m_t) - \mathcal{T}_t \\
&c_{t+1}^o = R_{t+1}k_{t+1} + \mathcal{T}_{t+1}
\end{aligned} \tag{B.15}$$

Here, we can write the Lagrangian function:

$\mathcal{L} = \ln c_t^y + \beta \ln c_{t+1}^o + \lambda_1(c_t^y + k_{t+1} - w_t(1-m_t) + \mathcal{T}_t) + \lambda_2(c_{t+1}^o - R_{t+1}k_{t+1} - \mathcal{T}_{t+1})$

The first order conditions:

$$\begin{aligned}
&\frac{\partial \mathcal{L}}{c_t^y} = \frac{1}{c_t^y} + \lambda_1 = 0 \\
&\frac{\partial \mathcal{L}}{c_{t+1}^o} = \frac{\beta}{c_{t+1}^o} + \lambda_2 = 0 \\
&\frac{\partial \mathcal{L}}{k_{t+1}} = \lambda_1 - \lambda_2 R_{t+1} = 0 \\
&\frac{\partial \mathcal{L}}{\lambda_1} = c_t^y + k_{t+1} - w_t(1-m_t) + \mathcal{T}_t = 0 \\
&\frac{\partial \mathcal{L}}{\lambda_2} = c_{t+1}^o - R_{t+1}k_{t+1} - \mathcal{T}_{t+1} = 0
\end{aligned} \tag{B.16}$$

The ratio of the first and the second partial derivative, combining with the third partial derivative result in

$$\frac{1}{\beta}\left(\frac{c_{t+1}^o}{c_t^y}\right) = R_{t+1} \iff c_{t+1}^o = c_t^y \beta R_{t+1}$$

Substituting to the fourth and the fifth partial derivative, we could solve for k_{t+1}.

$$\begin{aligned}
R_{t+1}k_{t+1} + \mathcal{T}_{t+1} &= (w_t(1-m_t) - \mathcal{T}_t - k_{t+1})\beta R_{t+1} \\
k_{t+1} + \frac{k_{t+1}}{\beta} + \frac{\mathcal{T}_{t+1}}{\beta R_{t+1}} &= w_t(1-m_t) - \mathcal{T}_t \\
k_{t+1} &= \left(\frac{\beta}{1+\beta}\right)\left(w_t(1-m_t) - \mathcal{T}_t - \frac{\mathcal{T}_{t+1}}{\beta R_{t+1}}\right)
\end{aligned} \tag{B.17}$$

For the firm decisions, the capital and labor employed will be the same as in the capital income subsidy case. substituting

$$R_{t+1} = \alpha z(E_t) k_{t+1}^{\alpha-1}$$

. we have nonlinear equation problem:

$$k_{t+1} = \left(\frac{\beta}{1+\beta}\right)\left(w_t(1-m_t) - \mathcal{T}_t - \frac{\mathcal{T}_{t+1}}{\alpha\beta z(E_t)k_{t+1}^{\alpha-1}}\right) \tag{B.18}$$

Because k_{t+1} cannot be solved analytically, we again rely on numerical solutions.

Table 5: System of Equations for Cobb-Douglas Lump Sum Transfers

$$k_{t+1} = \left(\frac{\beta}{1+\beta}\right)\left(w_t(1-m_t) - \mathcal{T}_t - \frac{\mathcal{T}_{t+1}}{\beta R_{t+1}}\right) \tag{B.19}$$

$$c_t^y = w_t(1-m_t) - \mathcal{T}_t - k_{t+1} \tag{B.20}$$

$$c_{t+1}^o = R_{t+1}k_{t+1} + \mathcal{T}_{t+1} \tag{B.21}$$

$$w_t = (1-\alpha)z(E_{t-1})k_t^\alpha \tag{B.22}$$

$$R_{t+1} = \alpha z(E_t)k_{t+1}^{\alpha-1} \tag{B.23}$$

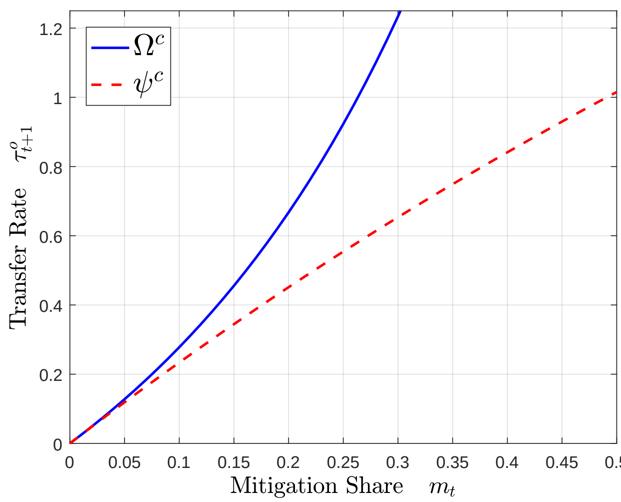
$$E_t = (1-\delta)E_{t-1} + \xi k_t - \gamma m_{t-1}w_{t-1} \tag{B.24}$$

$$z(E_t) = Ae^{-|E_t|} \tag{B.25}$$

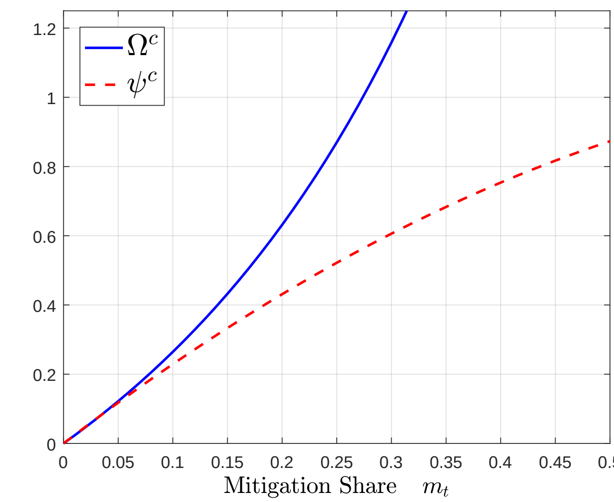
C Robustness

C.1 Varying Parameter ρ

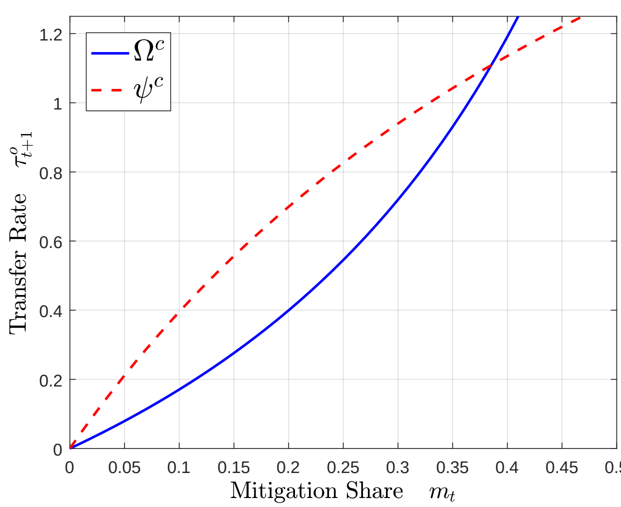
We try to simulate the capital income subsidy scheme under an easy and a difficult substitution of capital and labor.



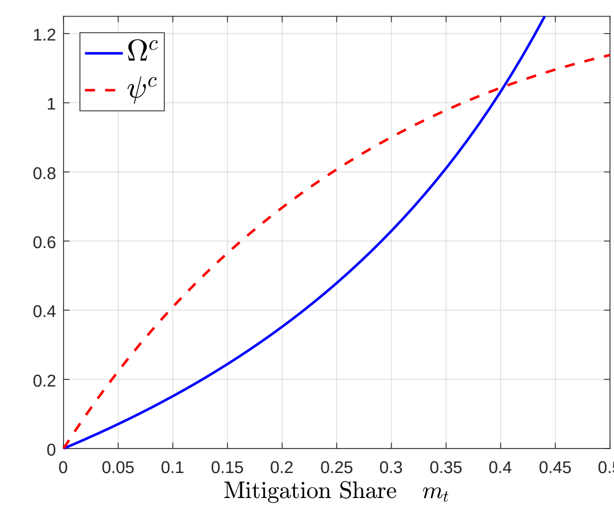
(a) Easy substitution with $\theta = 0.1$, $\rho = 0.1$, and $k_0 = 2$



(b) Difficult substitution with $\theta = 0.1$, $\rho = -0.1$, and $k_0 = 2$

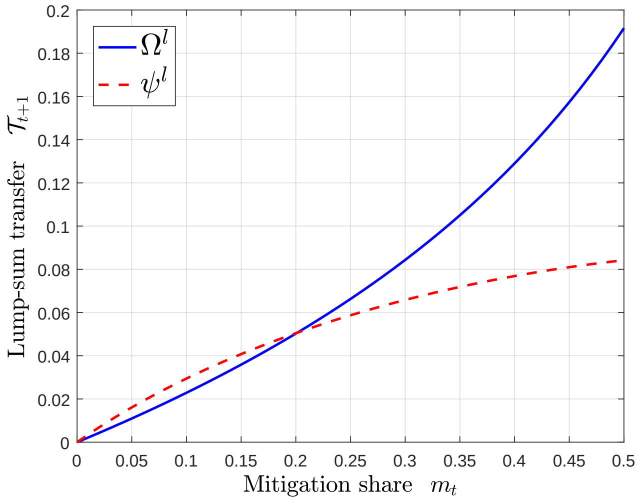


(c) Easy substitution with $\theta = -0.1$, $\rho = 0.1$, and $k_0 = 2$

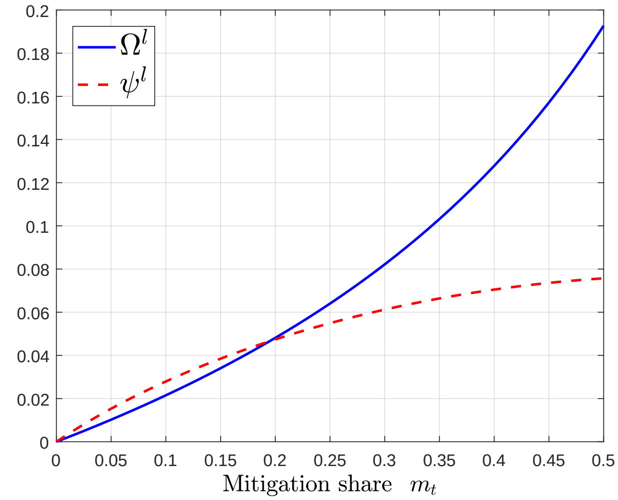


(d) Difficult substitution with $\theta = -0.1$, $\rho = -0.1$, and $k_0 = 2$

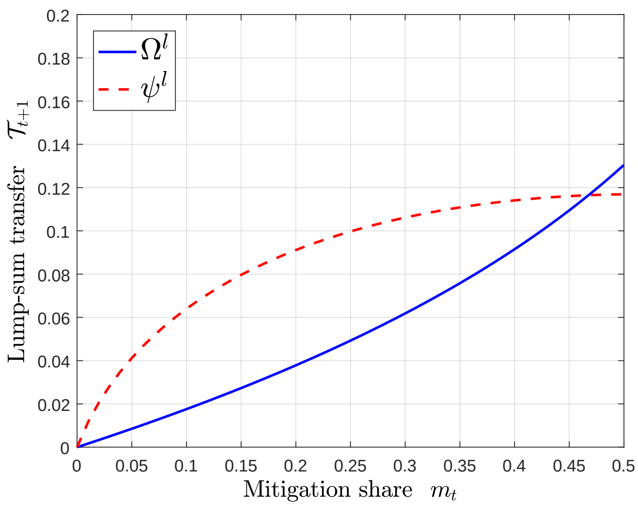
Figure 26: Capital Income Subsidy Scheme with Different θ and ρ



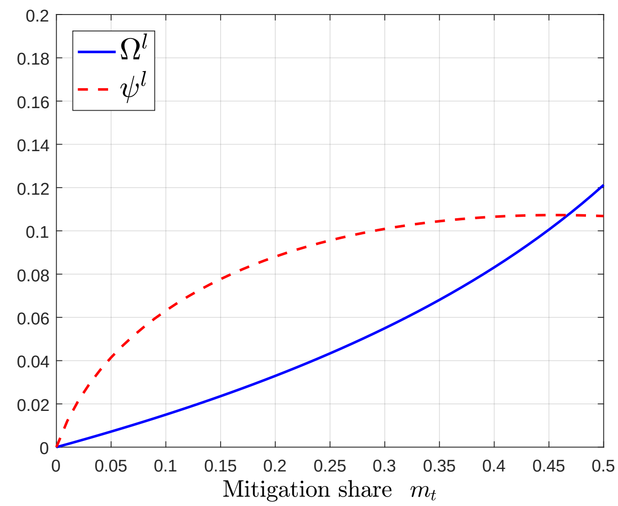
(a) Easy substitution with $\theta = 0.1$, $\rho = 0.1$, and $k_0 = 2$



(b) Difficult substitution with $\theta = 0.1$, $\rho = -0.1$, and $k_0 = 2$



(c) Easy substitution with $\theta = -0.1$, $\rho = 0.1$, and $k_0 = 2$



(d) Difficult substitution with $\theta = -0.1$, $\rho = -0.1$, and $k_0 = 2$

Figure 27: Lump-sum Transfer Scheme with Different θ and ρ