Vrije Universiteit Amsterdam

Master Thesis

Impact of Gas Subsidies on the Transition to Renewable Energy: A Mathematical Model Analysis

Author: Franka Stößer (2828395)

Supervisor: Gerard van der Meijden

A thesis submitted in fulfillment of the requirements for the VU Amsterdam Master of Spatial, Transport, and Environmental Economics

June 28, 2024

Abstract

This thesis explores the impact of gas subsidies on the transition from fossil fuels to renewable energy sources. While gas is often promoted as a transitional fuel, due to its lower emissions compared to coal, it can delay the full transition to renewable energy. The study examines how subsidies for gas decrease its production costs and change investments in renewables, affecting learning curves and cost reductions. Using a mathematical model, the research demonstrates that such subsidies extend the reliance on gas, delaying the achievement of zero-emission energy systems. Key findings highlight the need for careful policy design to avoid long-term dependency on fossil fuels.

Contents

1 Introduction **1**

2 Literature review **3**

3 Mathematical model **7**
- 3.1 Why Use a Mathematical Model? 7
- 3.2 Explanation and similarities to Pommeret et al. (2022) 7
- 3.3 Mathmatical model 8

4 Data **13**

5 Results **16**
- 5.1 Without Subsidy (s=0) 16
- 5.2 With Subsidy 17
  - 5.2.1 Low Subsidy (s=0.01) 17
  - 5.2.2 High Subsidy (s=0.02) 18
- 5.3 Cumulative emissions for different subsidy levels 18
- 5.4 Analysis 19
  - 5.4.1 Expected Results 20
  - 5.4.2 Discussion 20
  - 5.4.3 Potential Flaws and Limitations 20

6 Sensitivity Analysis **21**
- 6.1 Sensitivity analysis 1: 21
- 6.2 Sensitivity analysis 2: 21

7 Future research and conclusion **23**

References **25**

8 Appendix **27**
- 8.1 Steady-State Calculation 27
- 8.2 Calculate qc and qg 28
- 8.3 Sensitivity Analysis 29
  - 8.3.1 Sensitivity Analysis 1 29
  - 8.3.2 Sensitivity Analysis 2 31
- 8.4 Future research 33
  - 8.4.1 Shared Investments 33

8.4.2 Scarce gas resources 34

1 Introduction

One of the main focuses of tackling the climate crisis is a fast and low-emission transition from fossil fuels such as coal to zero-emission energy sources. In this context, gas is often considered a useful transition fuel when changing from coal to renewables due to its lower relative emissions compared to coal (IPCC, 2023). Initially, gas will mainly replace coal rather than renewables, as the market share of fossil fuels is currently higher, and its increased use is likely to reduce emissions in the short term. Therefore, many studies examine the impact of policies, such as subsidies and taxes, in order to increase the use of gas and, by doing so, reduce coal consumption. It is important to note that the first-best policy would be to introduce a carbon tax equal to the social cost of carbon and a subsidy for renewables production to capture future cost reductions due to learning (Coulomb et al., 2019). However, carbon taxes may face political barriers, for example due to opposition from industrial sectors and consumers who would bear the immediate costs (Jenkins, 2014). Consequently, subsidizing gas to lower coal use may be considered a second-best alternative to achieve emission reductions. The work of Coulomb et al. (2019) focuses for example on the optimal transition strategy in the power sector.

But when considering the long-term effect, these policies, aimed at reducing emissions, might support the opposite. Albeit they might reduce greenhouse gases at first, an increased use of gas could lead to longer dependence on fossil fuels and delay the full change to zero-emission energy sources of our society. Depending on how delayed this shift is, the overall emissions might be greater with gas as transition fuel than without.

There are two main reasons for this delay in the transition to renewable energy.

First, assuming there is a specific amount of Research and Development (R&D) resources at each point in time for energy sources. With pro-gas policies, a great amount of those resources will be focused on gas technology, that otherwise would be focused on renewable research. This reallocation of R&D resources has the consequence of delaying the maturing of renewable technology, and with this the possible change to zero energy sources (Acemoglu et al., 2023).

Second, another cause lies in the assumption of a fixed amount of investments for energy resources at each point of time. When considering gas as transition fuel, some of these investments will be reallocated from the development of renewables to that of gas. Over time, this reallocation increases the learning effect for gas technology and deters that of renewables when comparing to the situation without gas-supporting policies. But these learning effects are one of the main drivers of the reduction of energy-production costs. As a result, the production of renewable energies will stay costlier and that of gas will be cheaper. Therefore, the supply of gas stays higher and supply of renewables lower for a longer amount of time than without gas as intermediate fuel and the transition from fossil fuels to zero-emission energy sources will be delayed (McDonald and Schrattenholzer, 2001).

The topic of this thesis is connected to the latter, as it investigates the effect of a gas subsidy - representing the supporting policy for using gas as a transition fuel - on the production costs of gas and renewable energy. As the costs of gas decline through the subsidy, its market demand increases, while demand for renewables decreases. Consequently, investment in zero-emission energies diminishes, and the learning effect for renewable technology slows down. We analyze the potential impact of this on emissions over time, from the present until fossil fuels are fully phased out. Our research shows that with implementing a subsidy, the resulting greenhouse gases will be higher in the long-term than without the subsidy, as the increased use of gas leads to additional emissions that outweigh the reductions from using less coalin the short-term.Further, the delay in the transition to renewables, and the longer reliance on fossil fuels, can result in a longer use of not just gas, but also coal. The provided model aims to demonstrate the various possible outcomes based on the amount of subsidy provided.

The remaining part of this thesis is structured as follows. Section 2 presents a literature review of existing work researching the effect when using gas as transition fuel and of implementing a gas subsidy. Section 3 forms a mathematical model to analyze this effect by using optimal control theory. Section 4 presents data and assigns values to the used parameters in order to examine our model. Section 5 studies the outcome obtained from our model when implementing the parameter values. Section 6 performs a sensitivity analysis to determines how different parameter values affect the outcome and our conclusion. Finally, section 7 gives an overview of possible future research and an conclusion of this thesis.

2 Literature review

Researchers often disagree on whether the use of low-emission fossil fuels, such as gas, as intermediate fuels is optimal in the energy transition. While some literature highlights their benefits, others argue that these fuels hinder the shift to renewable energy.

The work of Coulomb et al. (2019) focuses on the optimal transition strategy in the power sector when using gas as a bridging fuel. Their findings indicate that, in the optimal extraction path, the social planner makes "full use of the large available gas and renewable capacities" (Coulomb et al., 2019).

However, one concern is that "investments in natural gas might crowd out investments in renewable alternatives" (Gürsan and de Gooyert, 2021). The use of gas can maintain dependency on fossil fuels, delaying the transition to renewable energy sources and potentially hindering global efforts to mitigate emissions. Whenever gas is chosen over renewable energy sources, its technology can mature and its infrastructure can expand, and the relative position of renewables declines against that of fossil fuels (Stößer, 2024). This phenomenon, known as carbon lock-in and studied by Unruh (2000), is making it increasingly difficult to fully switch to clean alternatives in the future. Further, due to the low gas prices, investments in renewable technology become less attractive. This effect is strengthened through continued support of gas. It must be kept in mind that "every energy source competes with each other to satisfy energy demand. If one technology becomes attractive, its advantage can be reinforced through the learning feedback" (Gürsan and de Gooyert, 2021).

Acemoglu et al. (2023) research the long-run consequences of using gas as a bridging fuel by examining its impact on R&D resource allocation between fossil fuels and renewables. Their findings suggest that policies supporting gas could divert crucial R&D efforts away from renewable technology, thereby delaying the transition to a green economy. The paper researches this by analyzing the possible effects of the shale gas revolution, which can be compared to the use of gas as transition fuel. When gas is supported by policies, R&D resources, otherwise focused on renewables, will be reallocated to gas technology. The paper finds that "the electricity sector has been sharply redirected away from renewable and green technology concurrently with the shale gas revolution in the United States" (Acemoglu et al., 2023).

Acemoglu et al. use a mathematical model to explore different outcomes based on various parameters, such as the rate of social time preference and the elasticity of substitution between fossil fuels and renewables. They show that under certain parameters, the natural gas boom can shift the economy from a green path to a fossil-fuel path, increasing long-term dependence on fossil fuels. For instance, with a low rate of social time preference, the shift towards natural gas significantly delays investment in renewables, leading to higher cumulative emissions over time. This analysis underscores the importance of carefully considering the long-term impacts of energy policies on innovation and the overall trajectory of the energy transition. This important paper supports my thesis by strengthening the findings that subsidizing gas as a transition fuel can delay the shift to zero-emission energy sources, while it uses a different approach.

One different approach in reasoning why low emission fuels as transition fuels are controversial is the impact of learning rates on energy technology. Learning curves of energy sources describe the cost reductions associated with accumulated experience, as analyzed by McDonald and Schrattenholzer (2001). Investments in gas can slow down the learning progress for renewables by redirecting financial resources. The authors highlight how the change in the learning curve for renewable technology can ultimately hinder their competitiveness. To research this effect further, we need more in depth research on, besides others, the amount of investment spend on gas and renewable energy technology. In a complementary study, Way et al. (2022) provide cost projections for different transition scenarios, from rapid to slow phase-outs of fossil fuels. Given those different trends, the possible costs for different energy sources, such as coal, gas and renewable energy are calculated. The data provided by the paper can be used for examining the impact of a gas-subsidy on the learning effect for gas and renewables. The cost changes can provide insights into necessary future investments for the different energy sectors, as well as how strong the learning effect changes over time when crowding out fossil fuels slower or faster (Way et al., 2022). Their analysis further underscores the economic benefits of accelerating the transition to renewables, which could be undermined by continued investments in gas.

The amount of investment in renewables is, among other things, dependent on innovation and energy efficiency. This means that, the higher the development of renewables through higher learning effect, the more increase investments in renewables. When more investments in one period are spent on renewables, the learning effects, and with this innovation and energy efficiency, increase. As a result, the share of investments spend on this energy sector will stay higher in the future (Shinwari et al., 2022).

One way to support the use of gas as a transition fuel are subsidies. This is also the tool examined in this thesis.

Energy subsidies are used by governments to influence energy markets and promote economic stability. They can be implemented as direct financial transfers, tax incentives, and price controls, and are applied to fossil fuels as well as renewable energy sources. The primary goal of energy subsidies is to ensure energy security and affordability, and, particularly for transition fuels such as gas, to support a faster outsourcing of the high emission fuel, coal. However, the role of subsidies in the transition to renewable energy has sparked significant debate among policymakers and researchers.

Rezai and van der Ploeg (2016) argue that while subsidies for renewable energy can have significant benefits, the absence of a global carbon tax might lead to suboptimal outcomes. Their mathematical model incorporates an optimization framework to assess the long-term impacts of renewable energy subsidies on the transition to a low-carbon economy. They find that without credible commitment, these subsidies can result in higher short-term fossil fuel use but ultimately reduce cumulative carbon emissions. The study highlights that the benefits of renewable subsidies could be undermined by the lack of a carbon tax, emphasizing the importance of considering long-term policy impacts on carbon emissions and the broader energy transition strategy.

Due to possible lowered production costs of fossil fuels, subsidies can influence market dynamics and investment decisions. According to Coady et al. (2019), fossil fuel subsidies 'are projected at $5.2 trillion (...) in 2017', which highlights their significant impact on the global energy landscape. These subsidies can delay the transition to renewable energy by making fossil fuels increasingly competitive against cleaner alternatives.

Empirical studies provide various results of the effectiveness of gas subsidies in reducing emissions. Studies like those by Lin and Ouyang (2014) suggest that the impact of subsidies on emissions depends on the situation. Their research shows that in some cases, subsidies for natural gas can reduce emissions by displacing more carbon-intensive fuels like coal.

On the other hand, Burniaux and Chateau (2014) find that removing fossil fuel subsidies could significantly reduce global carbon emissions by 2050. Their model incorporates various economic and policy scenarios to analyze the impact of subsidy removal on emissions and energy markets (Burniaux and Chateau, 2014). This highlights the complexity of subsidy policies and the need for a nuanced approach to evaluate their environmental impact.

The literature reviewed so far researches different aspects that are to be considered when examining the negative effects of gas as a bridging fuel and the implementation of a gas subsidy. While some studies do analyze the transition delay from fossil fuels to zero-emission energy sources and the impact of subsidies, they often do not examine the effect on the cost functions and learning progress of renewables using a comprehensive mathematical model. This thesis aims to fill this gap by specifically focusing on how a gas subsidy influences these economic dynamics, providing a detailed analysis through a mathematical framework.

Pommeret et al. (2021) discuss the role of critical raw materials (CRMs) in the transition to renewable energy systems. The authors develop a theoretical model to analyze the impact of CRM scarcity on the optimal path for energy transition, considering investments in green capital. Main parts of their model will be used in this thesis, such as using optimal control theory to solve the welfare maximization problem of the social planner.

Our analysis further complements that of Acemoglu et al. (2023) as they model technical change through productivity increases due to R&D, while we model it as cost declines through learning-by-doing. This distinction is crucial because it highlights different mechanisms through which gas subsidies can affect the transition to renewable energy sources. By examining the learning effects and cost dynamics, our work adds a nuanced perspective on the potential delays in transitioning to zero-emission fuels caused by gas subsidies.

The literature on the negative effects of transition fuels, particularly gas, highlights how their use can delay the transition to renewable energy. Coupled with the impact of subsidies, these dynamics can significantly alter the investment landscape and slow down the learning process critical for reducing renewable energy costs. While subsidies can play a crucial role in shaping energy markets, their design and implementation need to be carefully considered to ensure that they support the transition to a sustainable energy future. This thesis builds on the existing literature by examining the specific impact of gas subsidies on the transition from fossil fuels to renewables, providing new insights into the potential long-term environmental and economic consequences.

3 Mathematical model

3.1 Why Use a Mathematical Model?

In this thesis, we use a mathematical model to analyze the impact of a gas subsidy on the transition from fossil fuels to renewable energy sources. The choice of a mathematical approach is due to several key considerations:

A mathematical model helps us with capturing the complex interactions between economical, environmental, and technological factors involved. Our research further examines the effect of the policy over time, which requires a dynamic analysis that tracks changes in investments, costs, and emissions. The mathematical model provides the framework to simulate and analyze these dynamic processes. With it, we can also explore different scenarios, such as varying levels of subsidies. This flexibility enables us to assess the outcomes under different conditions that influence the transition process. Lastly, the model provides precise quantitative insights into how a subsidy affects the production costs of gas and renewable energy, the reallocation of investments, and the resulting emissions. These insights are crucial for developing evidence-based policy recommendations.

3.2 Explanation and similarities to Pommeret et al. (2022)

We have developed a mathematical model to investigate how a subsidy s for gas might delay the transition from fossil fuels to renewables, potentially leading to higher long-term emissions. The subsidy effectively lowers the production costs of gas in our model.

We expect the following outcome:

1. With reduced costs due to the subsidy, the demand for gas is expected to increase, whereas the demand for renewables will decrease.

2. As the demand for renewables decreases, investments I in renewable technology will also decrease.

3. The learning effect, crucial for reducing production costs of renewables, will slow down due to lower investments.

4. Consequently, gas will be used for a longer period compared to a scenario without the subsidy.

We use three energy sources in our model: Coal (c), Gas (g) and Renewables (K). The quantity of the first two energy source is denoted by $q_j,\ j \in \{c, g\}$. We also analyze the model over time t, $t \in [0, \infty]$

Comparison with Pommeret et al. (2022)

We use similarly the "green" capital $K(t)$ and the capital accumulation equation $\dot{K} = I - \delta K$ as Pommeret et al. (2022). However, our model differs in several key aspects:

Unlike their model, which has a carbon budget, we incorporate emission costs $(\omega_c q_c + \omega_g q_g)\tau$, where ω_c, ω_g are the emission factors and τ is the carbon tax.

We include two fossil resources, coal and gas, instead of just one fossil resource.

Our investment I does not rely either on a mineral input $m(t)$ or a backstop input $b(t)$, but is subject to convex costs.

3.3 Mathmatical model

Supply Functions

The supply functions for coal, gas, and renewables are given by:

$$q_c = \frac{p - \eta_c}{\alpha_c},\ q_g = \frac{p - \eta_g}{\alpha_g},\ q_r = \zeta K,\ q_c,\ q_g,\ q_r \geq 0$$

Where α_c, α_g are the slopes of the marginal cost functions for coal and gas respectively, and η_c, η_g are the vertical intercepts of the marginal cost functions. K represents the stock of renewable energy generators, producing in total ζK units of energy per unit of time.

The supply of renewables does not depend on the current energy-market price. At each point in time t, there is an existing stock of renewable energy generators that produce energy ζK with no generating costs. The production of new energy generators K depends on the investment I spent on it.

Cost functions

The cost functions for coal and gas are given by:

$$k_c = \frac{1}{2}\alpha_c q_c^2 + \eta_c q_c$$

$$k_g = \frac{1}{2}\alpha_g q_g^2 + \eta_g q_g$$

Energy Supply and Demand

The total energy supply q_S is:

$$q_S = q_c + q_g + \zeta K = \frac{p - \eta_c}{\alpha_c} + \frac{p - \eta_g}{\alpha_g} + \zeta K$$

The energy demand function q_D is:

$$p = x - \alpha_d q_D, \Rightarrow \; q_D = \frac{x - p}{\alpha_d}$$

Capital Accumulation

The capital accumulation equation is:

$$\dot{K} = I - \delta K$$

Where I is the investment and δ is the depreciation rate. The investment cost function is given by:

$$C(I) = \xi_0 I + \frac{\xi_1 + \mu K^{-\phi}}{2} I^2$$

Where μ is the learning rate, ξ_0 represents the baseline fixed cost of investment, and ξ_1 modifies the investment cost based on the amount of investment I. ϕ determines the elasticity of the learning rate with respect to the installed capacity.

This implies that the higher the stock of renewable energy generators K, the lower the slope of the marginal investment cost function.

Utility Function

The utility function per unit of energy supply q_S is:

$$u(q_S) = \beta ln(q_S)$$

Where β is a constant representing the weight or scaling factor of the utility derived from the energy supply.

Thus, we have:

$$u(q_S) = \beta ln(q_s) = \beta ln(\frac{p - \eta_c}{\alpha_c} + \frac{p - \eta_g}{\alpha_g} + \zeta K$$

Social Planner's Problem

The social planner aims to maximize the following function:

$$max \int_0^\infty (\beta ln(q_s) - (\xi_0 I + \frac{\xi_1 + \mu K^{-\phi}}{2} I^2) - \frac{1}{2}\alpha_c q_c^2 - \eta_c q_c - \frac{1}{2}\alpha_g q_g^2 - \eta_g q_g - (\omega_c q_c + \omega_g q_g)\tau) e^{\rho t} dt$$

$$\text{Subject to } \dot{K} = I - \delta K$$

Here, $(\omega_c q_c + \omega_g q_g)\tau$ represents the cost of emission per unit of coal and gas produced, ρ is the discount rate, and $e^{\rho t}$ is the discount factor.

Hamiltonian

We use the Hamiltonian to solve this problem:

$$\mathcal{H} = \beta ln(q_s) - (\xi_0 I + \frac{\xi_1 + \mu K^{-\phi}}{2} I^2) - \frac{1}{2}\alpha_c q_c^2 - \eta_c q_c - \frac{1}{2}\alpha_g q_g^2 - \eta_g q_g - (\omega_c q_c + \omega_g q_g)\tau + \lambda(I - \delta K)$$

Where λ is the co-state variable.

First Order Conditions

The first-order conditions are:

$$\frac{\partial \mathcal{H}}{\partial q_c} = \frac{\beta}{q_S} - \alpha_c q_c - \eta_c - \omega_c \tau = 0 \quad (1)$$

$$\frac{\partial \mathcal{H}}{\partial q_g} = \frac{\beta}{q_S} - \alpha_g q_g - \eta_g - \omega_g \tau = 0 \quad (2)$$

$$\frac{\partial \mathcal{H}}{\partial I} = -\xi_0 - (\xi_1 + \mu K^{-\phi}) I + \lambda = 0 \quad (3)$$

$$\frac{\partial \mathcal{H}}{\partial K} = \frac{\beta \zeta}{q_S} + \frac{1}{2}\mu\phi K^{-\phi-1} I^2 - \lambda\delta = -\dot{\lambda} + \rho\lambda \quad (4)$$

$$\frac{\partial \mathcal{H}}{\partial \lambda} = I - \delta K = 0 \quad (5)$$

We present the conditions here for an interior solution with $q_c,\ q_g > 0$, but will deal in the numerical analysis with corner solutions as well.

Steady-State Calculation

To calculate the steady states, $I_{steady},\ K_{steady}$, we need to determine the point in time when enough renewable energy generators K have been produced to meet the entire energy demand, thereby eliminating the need for fossil fuels. At this point, the investment in renewables stabilizes because the marginal investment cost function decreases with the increasing stock of renewable energy generators. Consequently, we will only need investments to cover the depreciation costs δ. Similarly, K will stabilize, with new generators being produced only to offset depreciation. However, it is important to note that the system is not immediately in a steady state once fossil fuels are fully phased out. Instead, the economy will be located on the saddle path leading to the fossil-free steady state.

In the steady state, we have $\dot{I} = 0$ and $\dot{K} = 0$ with $q_c = q_g = 0$. Therefore, the total energy supply q_S is then given by:

$$q_S = q_c + q_g + \zeta K = \zeta K$$

Using this, we derive the following equation:

$$I = \frac{1}{(\rho + \delta)(\xi_1 + \mu K^{-\phi})}(\frac{1}{2}\mu\phi K^{-\phi-1} I^2 - (\rho + \delta)\xi_0 + \frac{\beta}{K})$$

The calculations are elaborated in part 8.1 of the appendix.

We also know that $\dot{K} = I - \delta K = 0$, which implies:

$$I = \delta K$$

Substituting this into the previous equation, we get:

$$0 = (\frac{1}{2}\mu\phi\delta^2 - (\rho + \delta)\mu\delta)K^{-\phi+2} - (\rho + \delta)\xi_1\delta K^2 - (\rho + \delta)\xi_0 K + \beta$$

Solving this for K, we obtain the steady state value K_{steady}, representing the number of renewable energy generators required to fully displace fossil fuels. Once K_{steady} is found, we can determine I_{steady} using $I = \delta K$.

To calculate the steady states I_{steady} and K_{steady}, we need to derive a dynamical system in the variables q_c, q_g, K and I. This involves solving the system of differential equations that describe the evolution of these variables over time. The first-order conditions for the quantities of coal, gas, and renewables can be used to track their changes at any point in time t.

1. Derive the dynamical system in the variables q_c, q_g, K and I.
2. Start from the steady state values I_{steady} and K_{steady}.
3. Use time inversion to apply the system of differential equations. This helps construct the optimal trajectories of q_c, q_g, and K from the current state to the steady state.

By using this approach, we can simulate the dynamic processes and observe their evolution over time. This method allows us to explore different scenarios, such as varying levels of subsidies, and to assess the outcomes under different conditions. It provides precise quantitative insights into how a subsidy affects the production costs of gas and renewable energies, the reallocation of investments, and the resulting emissions. These insights are crucial for developing evidence-based policy recommendations.

Calculating q_c and q_g

Using the first-order conditions, we have:

$$q_c = \frac{\beta}{\alpha_g q_g + \eta_g + \omega_g \tau} - q_g - \zeta K$$

$$q_g = \frac{\beta}{\alpha_c q_c + \eta_c + \omega_c \tau} - q_c - \zeta K$$

We solve these equations for q_c and q_g in terms of K and the model parameters by using Wolfram Alpha.

The calculations are elaborated in part 8.2 of the appendix.

4 Data

In this section, we calculate the model's parameters using various data sources for current energy prices, supply, and more. This approach allows us to derive approximate values, making the model more realistic and aligned with real-world scenarios. However, it is important to note that these parameters are not exact, as the primary goal of this thesis is not to represent precise real-world outcomes but rather to provide an overview of the potential outcomes from implementing a gas subsidy.

To estimate the emission produced with and without the subsidy, we use the data from IPCC (2023), which provides average emissions for different fossil fuels, including coal and gas. These values are further crucial for calculating the cost of emissions per unit of coal and gas produced, represented in our model by the formula $(\omega_c q_c + \omega_g q_g)\tau$. Here, ω_c and ω_g represent the emissions per unit of coal and gas, respectively.

For our analysis, we choose anthracite as representative for coal and natural gas for gas, as they are among the most commonly used fossil fuels for energy production worldwide (Stößer, 2024). According to our sources, anthracite emits 353.88 kg of CO2 per MWh of energy, which equals $0.35388 kgCO_2$/kWh, while natural gas emits 201.96 kg of CO2 per MWh of energy, equating to $0.20196 kgCO_2$/kWh.

We incorporate the current global average price of carbon, which is €2.50 per ton as of 2024 (Normative, 2024), which is equal to 0.0025€/kgCO_2. This value is used to set τ = 0.0025€/kgCO_2 in our model.

Next, we consider the global energy demand. While our model does not directly use supply and demand mechanics, we use these data points to calculate realistic values for the parameters α_c, α_g and η_c, η_c. The global supply of natural gas is expected to reach approximately 4,100 billion cubic meters in 2023 (Union, 2023), and coal supply reached 8,634 million tonnes in 2022 (Agency, 2023). Converting these quantities, we find that 1 cubic meter of natural gas is approximately 10.55 kWh (Calculat.org, 2024), and 1 tonne of coal is about 8,141 kWh (Converter, 2024). Consequently, the current global supply is around 43,255 × 10^9 kWh for natural gas and 70,289 × 10^9 kWh for coal.

Regarding energy prices, as of September 2023, '(t)he world average price is 0.155 U.S. Dollar per kWh for household users and 0.151 U.S. Dollar per kWh for business users' (GlobalPetrolPrices.com, 2024). For our model, we set the price at time $t = 0$ to \$0.153 per kWh, which approximately equals €0.14 per kWh.

We further set the discount rate ρ to 0.02, the learning rate μ to 0.15 and the elasticity of the learning rate ϕ to 0.5.

With these data points, we can now assign values to the model parameters that approximately reflect a realistic scenario. Specifically:

$$
\begin{aligned}
p &= €0.14 \text{ per kWh} \\
\tau &= 0.0025\,€/\text{kgCO}_2 \\
\omega_c &= 0.35388\,\text{tCO}_2/\text{kWh} \approx 0.35\,\text{tCO}_2/\text{kWh} \\
\omega_g &= 0.20196\,\text{tCO}_2/\text{kWh} \approx 0.20\,\text{tCO}_2/\text{kWh} \\
\rho &= 0.02 \\
\mu &= 0.15 \\
\phi &= 0.5
\end{aligned}
$$

At time $t = 0$, we have:

$$
\begin{aligned}
q_c &\approx 70,289 \times 10^9\,\text{kWh} \\
q_g &\approx 43,255 \times 10^9\,\text{kWh}
\end{aligned}
$$

For simplicity, we set $\zeta = 5.2$. Additionally, we assume $\eta_c = 0.05$ and $\eta_g = 0.06$. From these, we derive:

$$
\begin{aligned}
q_c &= \frac{p - \eta_c}{\alpha_c} \Rightarrow \alpha_c = \frac{0.14 - 0.05}{70,289 \times 10^9} \Rightarrow \alpha_c \approx 1.2804 \times 10^{-15} \\
q_g &= \frac{p - \eta_g}{\alpha_g} \Rightarrow \alpha_g = \frac{0.14 - 0.06}{43,255 \times 10^9} \Rightarrow \alpha_g \approx 1.8495 \times 10^{-15}
\end{aligned}
$$

Given the extremely large values for q_c and q_g, which result in very small α_c and α_g, we simplify our model by scaling down q_c and q_g. This scaling ensures numerical stability and interpretability:

$$
\begin{aligned}
q_c &\approx 70.29\,\text{kWh} \\
q_g &\approx 43.26\,\text{kWh} \\
\alpha_c &= \frac{0.14 - 0.05}{70.29} \approx 0.00128 \\
\alpha_g &= \frac{0.14 - 0.06}{43.26} \approx 0.00185
\end{aligned}
$$

Using these scaled values ensures that our model remains computationally feasible while accurately capturing the essential dynamics of the system. It's important to interpret the results within the context of this scaling to maintain the relevance and accuracy of our findings.

We further assume:

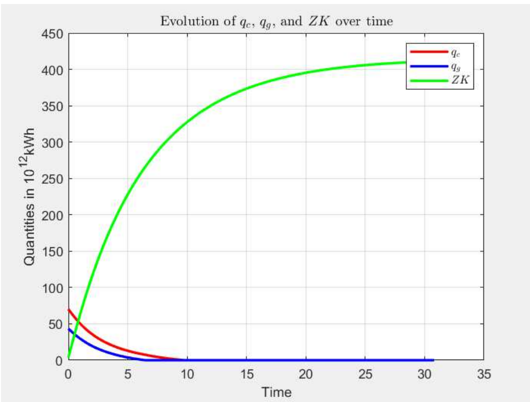
$$
\begin{aligned}
\delta &= 0.1 \\
\xi_0 &= 0.01 \\
\xi_1 &= 0.2 \\
\beta &= 16.5
\end{aligned}
$$

These values will be used to simulate different scenarios and analyze the potential impact of a gas subsidy on emissions and the transition to renewable energy.

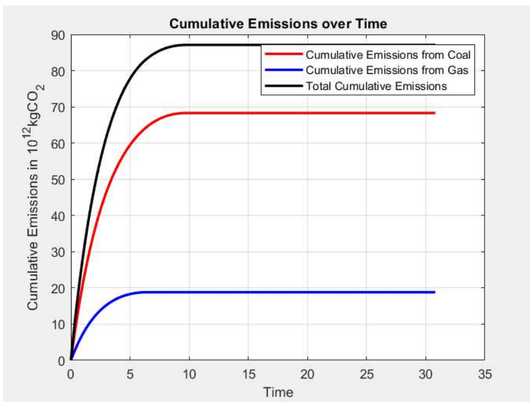
5 Results

The graphs illustrate the change in the use of different energy sources over time. To calculate the emission factors for coal and gas: coal emits $0.35kgCO_2$ per kWh and gas emits $0.20kgCO_2$ per kWh. We further calculate the quantities of coal and gas supplied through our MATLAB code. As we calculated earlier, the supply quantities for coal and gas were given as 70.29kWh and 43.26kWh respectively, instead of the original $70,289 \times 10^9$kWh and $43,255 \times 10^9$ kWh. Therefore, when interpreting the results, it is important to remember that we need to multiply the supplied quantities and cumulative emissions by $\times 10^{12}$ to reflect the actual values.

5.1 Without Subsidy (s=0)



(a) Quantities of kWh supplied over time



(b) Cumulative emissions over time

Figure 1: Model without a subsidy

In the scenario where there is no subsidy ($s = 0$), the additional cost parameter of gas, η_g, is higher than that of coal, η_c. As a result, gas will be phased out first, and coal will be used longer until we fully switch to renewables once their costs are low enough and supply high enough.

The area under q_c (red line) is $195,131.8 \times 10^9$, and the area under q_g (blue line) is $94,040.3 \times 10^9$. This means that the quantities of q_c and q_g produced are $195,131.8 \times 10^9$ and $94,040.3 \times 10^9$ kWh, respectively.

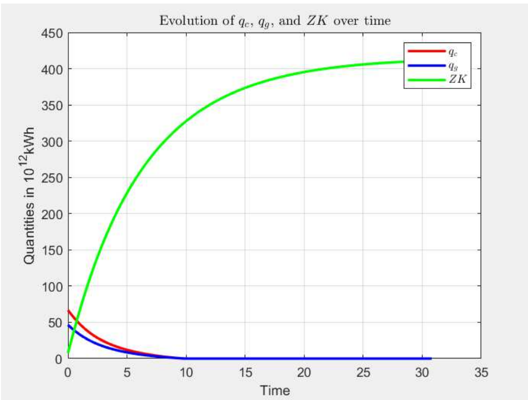
Using these quantities, we calculate the long-term emissions without a subsidy as follows:
- For coal, we have $195,131.8 \times 10^9 kWh * 0.35kgCO_2/kWh = 68,296.1 \times 10kgCO_2$.
- For gas, we have $94,040.3 \times 10^9 kWh * 0.20kgCO_2/kWh = 18,808.1 \times 10^9 kgCO_2$.

Together, this results in total emissions of $87,104.2 \times 10^9$ kg of CO_2.

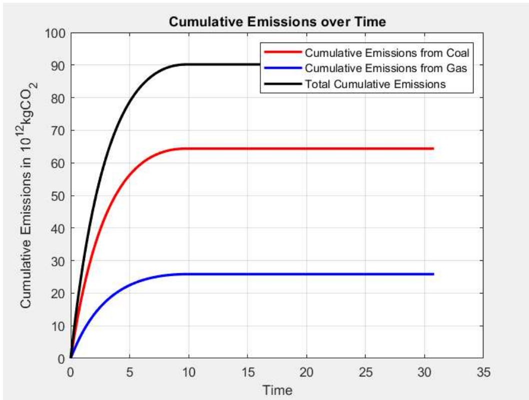
5.2 With Subsidy

Now we implement a subsidy s for gas in our model.

5.2.1 Low Subsidy (s=0.01)



(a) Quantities of kWh supplied over time



(b) Cumulative emissions over time

Figure 2: Model with a subsidy of s=0.01

If the subsidy is not high enough and the costs for gas are still higher than those for coal, gas will still be supplied less then coal, but it will be replaced at a later point than without the subsidy. Coal will also be phased out later, but a lower amount will be used at each point in time.

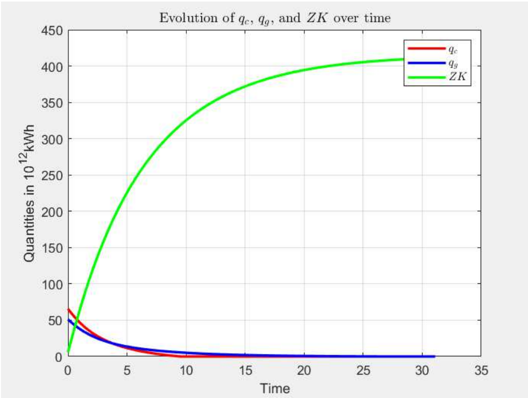
The area under q_c (red line) is $183,848.8 \times 10^9$, and the area under q_g (blue line) is $129,198.1 \times 10^9$. This means that the quantities of q_c and q_g produced are $183,848.8 \times 10^9$ and $129,198.1 \times 10^9$ kWh, respectively.

Using these quantities, we calculate the long-term emissions with a low subsidy as follows:

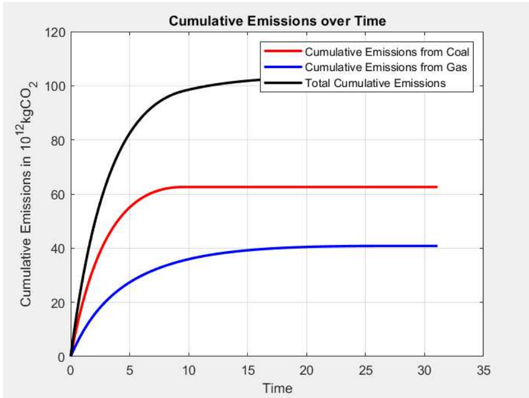
- For coal, we have $183,848.8 \times 10^9 kWh * 0.35 kgCO_2/kWh = 64,347.1 \times 10^9 kgCO_2$
- For gas, we have $129,198.1 \times 10^9 kWh * 0.20 kgCO_2/kWh = 25,839.6 \times 10^9 kgCO_2$.

Together, this results in total emissions of $90,186.7 \times 10^9$ kg of CO_2.

5.2.2 High Subsidy (s=0.02)



(a) Quantities of kWh supplied over time



(b) Cumulative emissions over time

Figure 3: Model with a subsidy of s=0.02

If the subsidy is high enough to significantly lower the costs of gas, coal will be replaced first, but again, later than without the subsidy, followed by gas until we only use renewable energy sources. A higher amount of gas will be used at each point in time then without the subsidy.

The area under q_c (red line) is $178,843.3 \times 10^9$, and the area under q_g (blue line) is $204,136.5 \times 10^9$. This means that the quantities of q_c and q_g produced are $178,843.3 \times 10^9$ and $204,136.5 \times 10^9$ kWh.

Using these quantities, we calculate the long-term emissions with the highest subsidy as follows:

- For coal, we have $178,843.3 \times 10^9 kWh * 0.35 kgCO_2/kWh = 62,595.1 \times 10^9 kgCO_2$
- For gas, we have $204,136.5 \times 10^9 kWh * 0.20 kgCO_2/kWh = 40,827.3 \times 10^9 kgCO_2$.

Together, this results in total emissions of $103,422.5 \times 10^9$kg of CO_2.

5.3 Cumulative emissions for different subsidy levels

The following graph displays the cumulative emissions for each subsidy level from $s \in [0, 0.02]$.

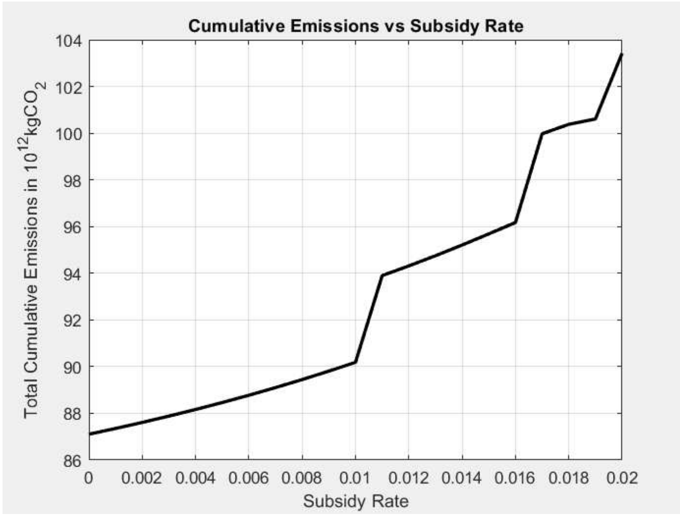


Figure 4: Cumulative emissions for $s \in [0, 0.02]$.

It shows that for each subsidy level chosen in our model, the total cumulative emissions will be higher compared to no gas subsidy.

When implementing a very small subsidy, such as $s = 0.001$, the subsidy can lead to a small decrease in cumulative emissions initially. At $t = 1$, the total emissions without a subsidy are $3,272.18 \times 10^9 kgCO_2$ and those for a subsidy of $s = 0.001$ are $3,268.46 \times 10^9 kgCO_2$. However, over time, this small decrease is outweighed by an increase in long-term emissions, as the reliance on gas and extended use of coal lead to higher overall emissions. This can be seen in the graph displaying the cumulative total emissions..

5.4 Analysis

Subsidy	$s = 0$	$s = 0.01$	$s = 0.02$
Long-term emissions in 10^9 **kg of** CO_2	87,104.2	90,186.7	103,422.5

Table 1: Long-term emissions in tons of CO_2 for different subsidy levels.

The results show that the level of impact of a subsidy on long-term CO_2 emissions is dependent on the subsidy level. But the impact will always be negative, as all subsidy levels increase the emissions. When implementing a very small subsidy, such as $s = 0.001$, the subsidy can lead to a small decrease in cumulative emissions initially. However, after short time, even then, emissions will increase in the long-term and outweigh the small decrease. Here is a more detailed discussion of the findings:

5.4.1 Expected Results

The results align with the expected outcomes in several ways:

Without a subsidy, more coal than gas is used, and gas will be replaced first due to coals lower production costs. This results in $87,104.2 \times 10^9$ kg of CO_2 emissions in the long-term. With the introduction of a low subsidy for gas, where the production costs for gas are still higher than that of coal, its phase-out is slightly delayed, and the supply of coal in the short-term is reduced. However, it still results in higher overall emissions ($90,186.7 \times 10^9$ kg of CO_2). At the highest subsidy level in our model, the extensive use of gas increases total emissions to $103,422.5 \times 10^9$ kg of CO_2. Theses scenarios highlight the potential negative impact of subsidies, where the increased reliance on gas and the longer reliance on coal and gas outweighs the benefits of reduced short-term coal use.

5.4.2 Discussion

The findings highlight several important considerations:

The analysis underscores that all levels of subsidies increase emissions compared to no subsidy. This increase is due to the extended reliance on fossil fuels, both coal and gas, delaying the full transition to renewable energy. While gas may initially replace coal and reduce its use, the reliance on gas extends the overall use of fossil fuels, leading to higher cumulative emissions. Policymakers should consider the balance between encouraging renewable energy use and the potential unintended consequences of increased reliance on fossil fuels like gas.

5.4.3 Potential Flaws and Limitations

Despite the insights provided, the analysis has several limitations:

The model makes different assumptions, such as fixed emission factors and linear cost functions. But in reality, these factors likely vary significantly over time and across different regions. We further do not account for exogenous factors that can significantly impact the results, such as technological advancements in renewable energy, geopolitical events affecting fuel supply, or changes in global energy demand. The practical implementation of subsidies involves administrative and political challenges. Our model assumes perfect implementation, which probably is not the case in a real-world scenario. Lastly, the model also assumes a fixed learning rate for renewables, which might not accurately capture the complex dynamics of technological advancement and market adoption.

Overall, the findings suggest that subsidies for gas lead to higher total emissions due to increased usage of gas and prolonged reliance on fossil fuels. Therefore, policy decisions regarding subsidies should consider the balance between encouraging renewable energy use and the potential unintended consequences of increased reliance on fossil fuels like gas.

6 Sensitivity Analysis

We conducted a sensitivity analysis to test the robustness of our conclusions by varying parameter values in our model and examining the resulting outcomes.

6.1 Sensitivity analysis 1:

In this analysis, we varied the following parameter values:

$$\tau = 0.004$$
$$\alpha_c = 0.0021$$
$$\omega_c = 0.4$$
$$\alpha_g = 0.0024$$
$$\omega_g = 0.22$$
$$\beta = 17$$

These changes include:

1. An increase in the slopes of the marginal cost functions ($\alpha_c,\ \alpha_g$)
2. An increase in the carbon tax (τ)
3. Adjustments in the emission factors($\omega_c,\ \omega_g$)
4. And an increase in β, representing the weight of the utility

6.2 Sensitivity analysis 2:

For the second analysis, we varied these parameter values:

$$\delta = 0.12$$
$$\rho = 0.015$$
$$\mu = 0.2$$
$$\xi_0 = 0.13$$
$$\xi_1 = 0.15$$
$$\phi = 0.6$$
$$\zeta = 5.4$$

These changes include:

1. An increase in the depreciation rate (δ)
2. An increase in the learning rate (μ)
3. An increase in the baseline fixed cost of investment (ξ_0)
4. An increase in the elasticity of the learning effect (ϕ)
5. An increase in the production efficiency of renewable energy generators (ζ)
6. A decrease in the discount rate (ρ)
7. A decrease in the modifier of the investment cost (ξ_1)

Each parameter was adjusted by a different amount to assess the impact on the model's outcomes.

The detailed results of the sensitivity analysis are provided in part 8.3 of the appendix. The analysis indicates that while varying parameter values can influence the magnitude of the emissions increase, the overall conclusion remains consistent: subsidies for gas lead to higher total emissions compared to scenarios without a subsidy.

This sensitivity analysis confirms the robustness of our initial findings. Even when accounting for a range of parameter variations, the fundamental conclusion holds. However, it is important to note that the extent of the emissions increase is sensitive to the specific parameter values, highlighting the need for careful calibration of policy instruments such as subsidies and taxes to achieve desired environmental outcomes.

7 Future research and conclusion

This thesis represents a basic model to analyze the potential outcome of a gas subsidy on the transition from fossil fuels to renewable energy sources. The model aims to provide an initial overview, considering the effect of the subsidy on the learning curve of renewables.

Further research can elaborate on this by including several additional factors to make the model more comprehensive and reflective of real-world scenarios.

One of these factors could be the possibility of shared investments between gas and renewables. The subsidy will lead to increased investments in the gas sector, and with this, decreased investments in renewable technology. Including this aspect would likely show that the learning effect for renewables decreases to an even larger extend than it does in our model, further illustrating the potentially adverse impacts of gas subsidies.

Another extension could involve accounting for the fact that gas is a finite resource, and that its extraction will become costlier over time as its existing stock decreases. Incorporating a dynamic cost function for gas extraction would likely lead to a faster phase-out of gas, which might mitigate some of the negative effects of the subsidy observed in the current model. This would provide a more nuanced understanding of how subsidies might interact with resource depletion dynamics.

The shared investments between gas and renewables and a dynamic cost function for gas extraction are elaborated in part 8.4 of the appendix.

Further, different aspects can be considered to make the model more realistic, such as an increasing demand for energy over time. This would help aligning it with current energy consumption trends, offering insights into how subsidies might perform under growing energy demands.

Future research could also explore the impact of technological advancements in renewable energy, which can significantly change the cost functions and efficiency of renewable energy sources.

Lastly, interactions between a gas subsidy and other policies, such as taxes, could be analyzed. This can help for a more effective policy framework.

We did not include these extensions in the research in order to keep the model as simple as possible by still being able to show the desired result.

The results of our model illustrate that all levels of subsidies for gas lead to higher emissions compared to scenarios without a subsidy. This finding challenges the notion that subsidies can effectively mitigate the climate crisis by lowering greenhouse gases. Instead, our model shows that, instead of mitigating emissions, the policy exacerbates the issue.

As stated before, the model presented in this thesis is not calibrated with exact real-world data for each parameter but serves as a theoretical framework to explore various outcomes when implementing a gas subsidy. This theoretical approach provides insights into the complexities and potential pitfalls of subsidy policies.

Key findings are that gas subsidies, regardless of the level, extend the reliance on fossil fuels and delay the transition to renewable energy sources. This extended reliance results in higher cumulative emissions over time, contrary to the policy's intended environmental benefits.

Policymakers must exercise caution in setting subsidy levels, taking into account all influencing factors. Accurately calculating the parameters and carefully determining the subsidy amount is essential. While subsidies for gas might seem like a viable short-term solution for reducing coal use, they ultimately hinder the transition to zero-emission energy sources by promoting prolonged use of fossil fuels.

A subsidy alone can be counterproductive; it needs to be combined with other policies. Instead of solely relying on gas subsidies, policymakers should consider implementing a carbon tax equal to the social cost of carbon and providing subsidies for renewable energy production. This combined approach, as suggested in the introduction, could better support the transition to a sustainable energy future by directly incentivizing the adoption of renewables and discouraging fossil fuel use.

In conclusion, while a gas subsidy has the potential to temporarily assist in the transition to zero-emission energy, it requires meticulous planning and precise implementation. Policymakers must balance the immediate economic benefits with the long-term environmental impact, ensuring that subsidies are designed to genuinely contribute to emission reductions rather than inadvertently promoting higher emissions.

Future research can extend this model by incorporating shared investments, resource scarcity, increasing energy demand, technological advancements, and policy interactions to provide a more comprehensive understanding of the impacts of gas subsidies. This ongoing research is crucial for developing robust policies that effectively support the transition to a sustainable energy future.

References

D. Acemoglu, P. Aghion, L. Barrage, and D. Hémous. Climate change, directed innovation, and energy transition: The long-run consequences of the shale gas revolution. Technical Report 31657, National Bureau of Economic Research, September 2023.

International Energy Agency. Coal market update july 2023: Supply, 2023. URL https://prod.iea.org/reports/coal-market-update-july-2023/supply. Accessed: 2024-06-10.

J.-M. Burniaux and J. Chateau. Greenhouse gases mitigation potential and economic efficiency of phasing-out fossil fuel subsidies. *Energy Economics*, 40:71–88, December 2014.

Calculat.org. Gas consumption calculator, 2024. URL https://www.calculat.org/en/energy-fuel/gas-consumption/. Accessed: 2024-06-10.

D. Coady, I. W. H. Parry, N. P. Le, and B. Shang. Global fossil fuel subsidies remain large: An update based on country-level estimates. *International Monetary Fund*, 2019/089, May 2019.

Kyle's Converter. Tons of coal equivalent to kilowatt-hours, 2024. URL https://www.kylesconverter.com/energy,-work,-and-heat/tons-of-coal-equivalent-to-kilowatt--hours. Accessed: 2024-06-10.

R. Coulomb, O. Lecuyer, and A. Vogt-Schilb. Optimal transition from coal to gas and renewable power under capacity constraints and adjustment costs. *Environmental and Resource Economics*, 73:557–590, July 2019.

GlobalPetrolPrices.com. Electricity prices, 2024. URL https://www.globalpetrolprices.com/electricity_prices/. Accessed: 2024-06-09.

C. Gürsan and V. de Gooyert. The systemic impact of a transition fuel: Does natural gas help or hinder the energy transition? *Energy Research & Social Science*, 68:101728, March 2021.

IPCC. Carbon dioxide emissions factor. *IPCC - Emission Factor Database (2023) – with minor processing by Our World in Data*, 2023. URL https://ourworldindata.org/grapher/carbon-dioxide-emissions-factor?country=Anthracite~Bitumen~Crude+Oil~Natural+Gas#sources-and-processing. Accessed: 2024-06-05.

J. Jenkins. Real-world barriers to carbon pricing, September 2014. URL https://thebreakthrough.org/issues/energy/real-world-barriers-to-carbon-pricing. Accessed: 2024-06-24.

B. Lin and X. Ouyang. A revisit of fossil-fuel subsidies in china: challenges and opportunities for energy price reform. *Energy Economics*, 82:124–134, June 2014.

A. McDonald and L. Schrattenholzer. Learning rates for energy technologies. *Energy Policy*, 29(4):255–261, April 2001.

Normative. Global carbon pricing & cop26: a primer. *Normative*, 2024. URL https://normative.io/insight/global-carbon-pricing-cop26/#:~:text=The%20current%20global%20average%20price,net%20zero%20emissions%20by%202050. Accessed: 2024-06-13.

A. Pommeret, F. Ricci, and K. Schubert. Critical raw materials for the energy transition. *European Economic Review*, 141:103949, December 2021.

A. Rezai and F. van der Ploeg. Second-best renewable subsidies to de-carbonize the economy: Commitment and the green paradox. *Environmental and Resource Economics*, 66:409–434, November 2016.

R. Shinwari, Y. Wang, A. H. Payab, J. Kubiczek, and H. Dordüncü. What drives investment in renewable energy resources? evaluating the role of natural resources volatility and economic performance for china. *Resources Policy*, 77:102738, April 2022.

F. Stößer. Navigating the transitional energy landscape: What to consider when using oil and gas as transition fuels in the shift from coal to renewables. 2024.

International Gas Union. Global gas report 2023 edition, 2023. URL https://igu.org/resources/global-gas-report-2023-edition/. Accessed: 2024-06-10.

G. C. Unruh. Understanding carbon lock-in. *Energy Policy*, 28(12):817–830, December 2000.

R. Way, M. C. Ives, P. Mealy, and J. D. Farmer. Empirically grounded technology forecasts and the energy transition. *Joule*, 6(9):2057–2082, September 2022.

8 Appendix

8.1 Steady-State Calculation

We have in (4) = $-\dot{\lambda} + \rho\lambda$ instead of just $-\dot{\lambda}$, as we didn't include the discount factor in the Hamiltonian and therefore add it here.

We get with (3):

$$\lambda = \xi_0 + (\xi_1 + \mu K^{-\phi})I$$

And form the derivative by t:

$$\dot{\lambda} = (\xi_1 + \mu K^{-\phi})\dot{I} - \phi\mu I K^{-1-\phi}\dot{K}$$

Putting this in (4) results in:

$$\frac{\beta\zeta}{q_S} + \frac{1}{2}\mu\phi K^{-\phi-1}I^2 = -(\xi_1 + \mu K^{-\phi})\dot{I} + \phi\mu I K^{-1-\phi}\dot{K} + (\rho+\delta)(\xi_0 + (\xi_1 + \mu K^{-\phi})I)$$

$$\Rightarrow (\xi_1 + \mu K^{-\phi})\dot{I} = \phi\mu I K^{-1-\phi}(I - \delta K) + (\rho+\delta)(\xi_0 + (\xi_1 + \mu K^{-\phi})I) - \frac{\beta\zeta}{q_S} - \frac{1}{2}\mu\phi K^{-\phi-1}I^2$$

$$\Rightarrow \dot{I} = \frac{1}{(\xi_1 + \mu K^{-\phi})}(\phi\mu I K^{-1-\phi}(I - \delta K) + (\rho+\delta)(\xi_0 + (\xi_1 + \mu K^{-\phi})I) - \frac{\beta\zeta}{q_S} - \frac{1}{2}\mu\phi K^{-\phi-1}I^2)$$

Therefore, in the steady state, we have $\dot{I} = 0$ and $\dot{K} = 0$ and $q_c, q_g = 0$.
It is then $q_S = q_c + q_g + \zeta K = \zeta K$

$$\dot{I} = \frac{1}{(\xi_1 + \mu K^{-\phi})}(\phi\mu I K^{-1-\phi}(I - \delta K) + (\rho+\delta)(\xi_0 + (\xi_1 + \mu K^{-\phi})I) - \frac{\beta\zeta}{q_S} - \frac{1}{2}\mu\phi K^{-\phi-1}I^2)$$

$$\Rightarrow 0 = (\rho+\delta)(\xi_0 + (\xi_1 + \mu K^{-\phi})I) - \frac{\beta}{K} - \frac{1}{2}\mu\phi K^{-\phi-1}I^2$$

$$\Rightarrow 0 = \frac{1}{2}\mu\phi K^{-\phi-1}I^2 - (\rho+\delta)(\xi_1 + \mu K^{-\phi})I - (\rho+\delta)\xi_0 + \frac{\beta}{K}$$

$$\Rightarrow I = \frac{1}{(\rho+\delta)(\xi_1+\mu K^{-\phi})}(\frac{1}{2}\mu\phi K^{-\phi-1}I^2 - (\rho+\delta)\xi_0 + \frac{\beta}{K})$$

We further know $\dot{K} = I - \delta K = 0 \Rightarrow I = \delta K$

$$\Rightarrow 0 = \frac{1}{2}\mu\phi K^{-\phi-1}(\delta K)^2 - (\rho+\delta)(\xi_1+\mu K^{-\phi}))\delta K - (\rho+\delta)\xi_0 + \frac{\beta}{K}$$

$$\Rightarrow 0 = \frac{1}{2}\mu\phi\delta^2 K^{-\phi+1} - (\rho+\delta)\xi_1\delta K - (\rho+\delta)\mu\delta K^{-\phi+1} - (\rho+\delta)\xi_0 + \frac{\beta}{K}$$

$$\Rightarrow 0 = (\frac{1}{2}\mu\phi\delta^2 - (\rho+\delta)\mu\delta)K^{-\phi+1} - (\rho+\delta)\xi_1\delta K - (\rho+\delta)\xi_0 + \frac{\beta}{K}$$

$$\Rightarrow 0 = K((\frac{1}{2}\mu\phi\delta^2 - (\rho+\delta)\mu\delta)K^{-\phi+1} - (\rho+\delta)\xi_1\delta K - (\rho+\delta)\xi_0) + \beta$$

$$\Rightarrow 0 = (\frac{1}{2}\mu\phi\delta^2 - (\rho+\delta)\mu\delta)K^{-\phi+2} - (\rho+\delta)\xi_1\delta K^2 - (\rho+\delta)\xi_0 K + \beta$$

8.2 Calculate qc and qg

We get with the first-order conditions (1) and (2):

$$\frac{\partial\mathcal{H}}{\partial q_c} = \frac{\beta}{q_S} - \alpha_c q_c - \eta_c - \omega_c\tau = 0$$

$$\Rightarrow \frac{\beta}{q_c+q_g+\zeta K} = \alpha_c q_c + \eta_c + \omega_c\tau$$

$$\Rightarrow q_c = \frac{\beta}{\alpha_g q_g + \eta_g + \omega_g\tau} - q_g - \zeta K$$

$$\frac{\partial\mathcal{H}}{\partial q_g} = \frac{\beta}{q_S} - \alpha_g q_g - \eta_g - \omega_g\tau = 0$$

$$\Rightarrow \frac{\beta}{q_c+q_g+\zeta K} = \alpha_g q_g + \eta_g + \omega_g\tau$$

$$\Rightarrow q_g = \frac{\beta}{\alpha_c q_c + \eta_c + \omega_c\tau} - q_c - \zeta K$$

We solve these equations for q_c and q_g using Wolfram Alpha and get:

$$q_{c1} = \left(-\frac{1}{2\alpha_c(\alpha_c+\alpha_g)}\right)\left(-\alpha_c(-2\eta_c+\eta_g+(-2\omega_c+\omega_g)\tau)+\alpha_g(\eta_c+\omega_c\tau+\alpha_c K\zeta)\right.$$
$$\left.+\sqrt{\begin{array}{c}\alpha_g^2(\eta_c+\omega_c\tau)^2+2\alpha_c\alpha_g(2\alpha_g\beta+(\eta_c+\omega_c\tau)(\eta_g+\omega_g\tau)-\alpha_g K(\eta_c+\omega_c\tau)\zeta)\\+\alpha_c^2(4\alpha_g\beta+(\eta_g+\omega_g\tau)^2+\alpha_g K\zeta(-2(\eta_g+\omega_g\tau)+\alpha_g K\zeta))\end{array}}\right)$$

$$q_{c2} = \left(\frac{1}{2\alpha_c(\alpha_c+\alpha_g)}\right)\left(\alpha_c(-2\eta_c+\eta_g+(-2\omega_c+\omega_g)\tau)-\alpha_g(\eta_c+\omega_c\tau+\alpha_c K\zeta)\right.$$
$$\left.+\sqrt{\begin{array}{c}\alpha_g^2(\eta_c+\omega_c\tau)^2+2\alpha_c\alpha_g(2\alpha_g\beta+(\eta_c+\omega_c\tau)(\eta_g+\omega_g\tau)-\alpha_g K(\eta_c+\omega_c\tau)\zeta)\\+\alpha_c^2(4\alpha_g\beta+(\eta_g+\omega_g\tau)^2+\alpha_g K\zeta(-2(\eta_g+\omega_g\tau)+\alpha_g K\zeta))\end{array}}\right)$$

$$q_{g1} = \left(-\frac{1}{2\alpha_g(\alpha_c+\alpha_g)}\right)\left(-\alpha_g(\eta_c-2\eta_g+(\omega_c-2\omega_g)\tau)+\alpha_c(\eta_g+\omega_g\tau+\alpha_g K\zeta)\right.$$
$$\left.+\sqrt{\begin{array}{c}\alpha_g^2(\eta_c+\omega_c\tau)^2+2\alpha_c\alpha_g(2\alpha_g\beta+(\eta_c+\omega_c\tau)(\eta_g+\omega_g\tau)-\alpha_g K(\eta_c+\omega_c\tau)\zeta)\\+\alpha_c^2(4\alpha_g\beta+(\eta_g+\omega_g\tau)^2+\alpha_g K\zeta(-2(\eta_g+\omega_g\tau)+\alpha_g K\zeta))\end{array}}\right)$$

$$q_{g2} = \left(\frac{1}{2\alpha_g(\alpha_c+\alpha_g)}\right)\left(-\alpha_c(\eta_g+\omega_g\tau)+\alpha_g(\eta_c-2\eta_g+\omega_c\tau-2\omega_g\tau-\alpha_c K\zeta)\right.$$
$$\left.+\sqrt{\begin{array}{c}\alpha_g^2(\eta_c+\omega_c\tau)^2+2\alpha_c\alpha_g(2\alpha_g\beta+(\eta_c+\omega_c\tau)(\eta_g+\omega_g\tau)-\alpha_g K(\eta_c+\omega_c\tau)\zeta)\\+\alpha_c^2(4\alpha_g\beta+(\eta_g+\omega_g\tau)^2+\alpha_g K\zeta(-2(\eta_g+\omega_g\tau)+\alpha_g K\zeta))\end{array}}\right)$$

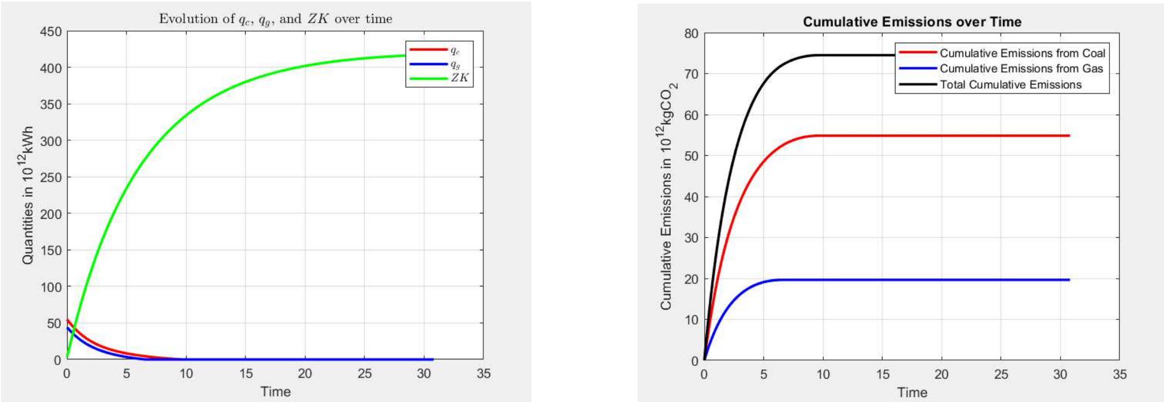
8.3 Sensitivity Analysis

We set the subsidy again to $s = 0,\ 0.005,\ 0.01,\ 0.015$

8.3.1 Sensitivity Analysis 1

We now use $\omega_c = 0.4$ and $\omega_g = 0.22$

s=0:



(a) Quantities of kWh supplied over time (b) Cumulative emissions over time

Figure 5: Model without a subsidy

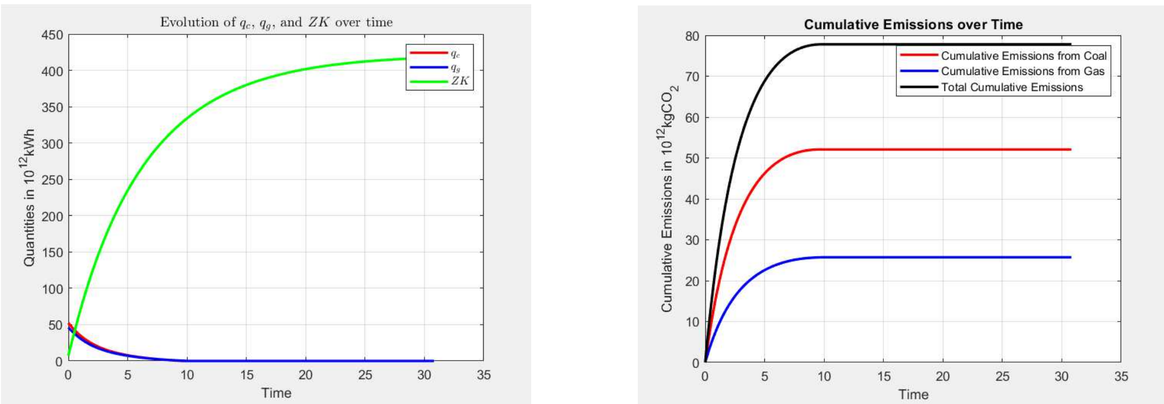
The area under q_c (red line) is $137,091.5 \times 10^9$, and the area under q_g (blue line) is $89,318.4 \times 10^9$. This means that the quantities of q_c and q_g produced are $137,091.5 \times 10^9$ and $89,318.4 \times 10^9$ kWh, respectively.

Using these quantities, we calculate the long-term emissions without a subsidy as follows:

- For coal, we have $137,091.5 \times 10^9 kWh * 0.4 kgCO_2/kWh = 54,836.6 \times 10^9 kgCO_2$.
- For gas, we have $89,318.4 \times 10^9 kWh * 0.22 kgCO_2/kWh = 19,650.0 \times 10^9 kgCO_2$.

Together, this results in total emissions of $74,486.6 \times 10^9$ kg of CO_2.

s=0.01:



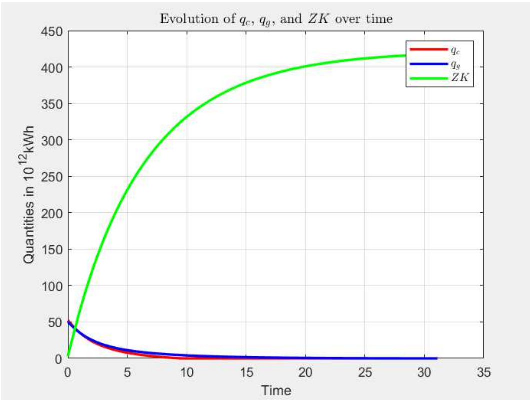
(a) Quantities of kWh supplied over time (b) Cumulative emissions over time

Figure 6: Model with a subsidy of s=0.01

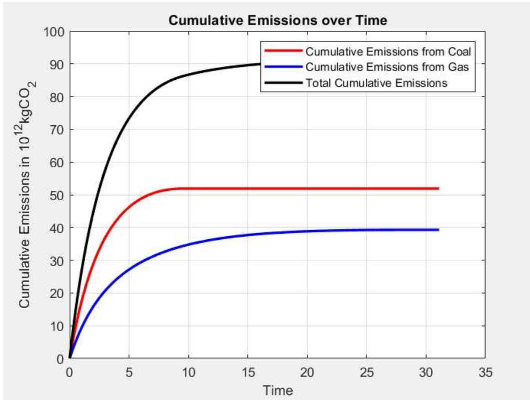
The area under q_c (red line) is $130,205.9 \times 10^9$, and the area under q_g (blue line) is $116,877.3 \times 10^9$. This means that the quantities of q_c and q_g produced are $130,205.9 \times 10^9$ and $116,877,3 \times 10^9$ kWh, respectively.

Using these quantities, we calculate the long-term emissions without a subsidy as follows:
- For coal, we have $130,205.9 \times 10^9 kWh * 0.4 kgCO_2/kWh = 52,082.3 \times 10^9 kgCO_2$.
- For gas, we have $116,877.3 \times 10^9 kWh * 0.22 kgCO_2/kWh = 25,713,0 \times 10^9 kgCO_2$.
Together, this results in total emissions of $77,795.4 \times 10^9$ kg of CO_2.

s=0.02:



(a) Quantities of kWh supplied over time



(b) Cumulative emissions over time

Figure 7: Model with a subsidy of s=0.02

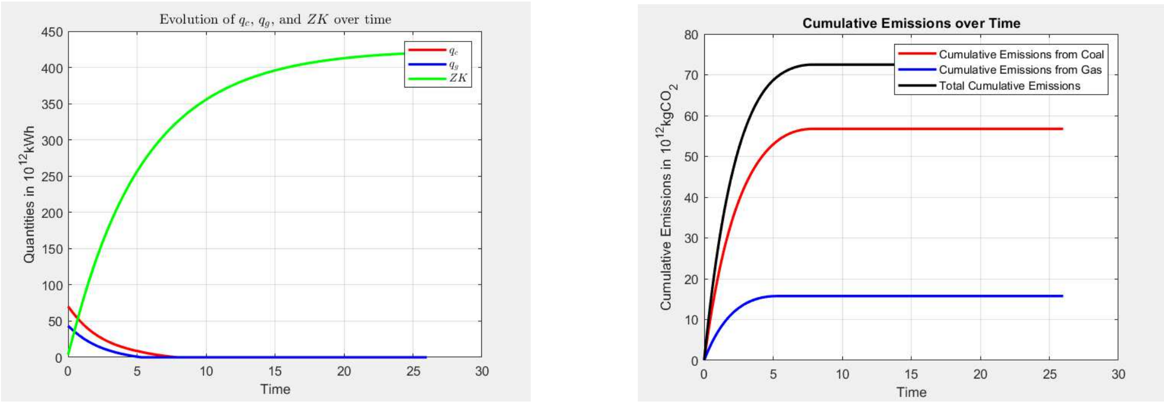
The area under q_c (red line) is $129,724.7 \times 10^9$, and the area under q_g (blue line) is $178,558.7 \times 10^9$. This means that the quantities of q_c and q_g produced are $129,724.7 \times 10^9$ and $178,558.7 \times 10^9$ kWh, respectively.

Using these quantities, we calculate the long-term emissions without a subsidy as follows:
- For coal, we have $129,724.7 \times 10^9 kWh * 0.4 kgCO_2/kWh = 51,889.9 \times 10^9 kgCO_2$.
- For gas, we have $178,558.7 \times 10^9 kWh * 0.22 kgCO_2/kWh = 39,282.9 \times 10^9 kgCO_2$.
Together, this results in total emissions of $91,172.8 \times 10^9$ kg of CO_2.

8.3.2 Sensitivity Analysis 2

We use now again $\omega_c = 0.35$ and $\omega_g = 0.20$

s=0:



(a) Quantities of kWh supplied over time
(b) Cumulative emissions over time

Figure 8: Model without a subsidy

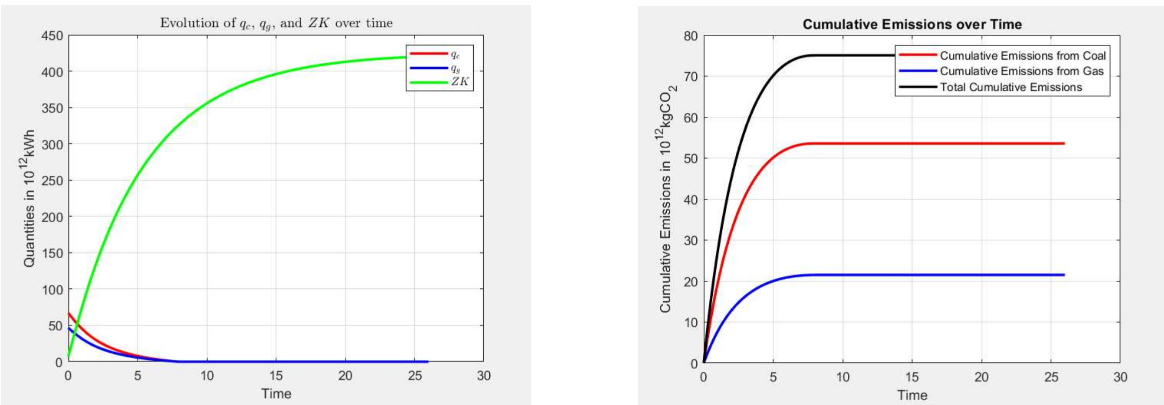
The area under q_c (red line) is $162,123.0 \times 10^9$, and the area under q_g (blue line) is $78,760.0 \times 10^9$. This means that the quantities of q_c and q_g produced are $162,123.0 \times 10^9$ and $78,760.0 \times 10^9$ kWh, respectively.

Using these quantities, we calculate the long-term emissions without a subsidy as follows:

- For coal, we have $162,123. \times 10^9 kWh * 0.35 kgCO_2/kWh = 56,743.1 \times 10^9 kgCO_2$.
- For gas, we have $78,760.0 \times 10^9 kWh * 0.20 kgCO_2/kWh = 15,750.0 \times 10^9 2 kgCO_2$.

Together, this results in total emissions of $72,495.1 \times 10^9$ kg of CO_2.

s=0.01:



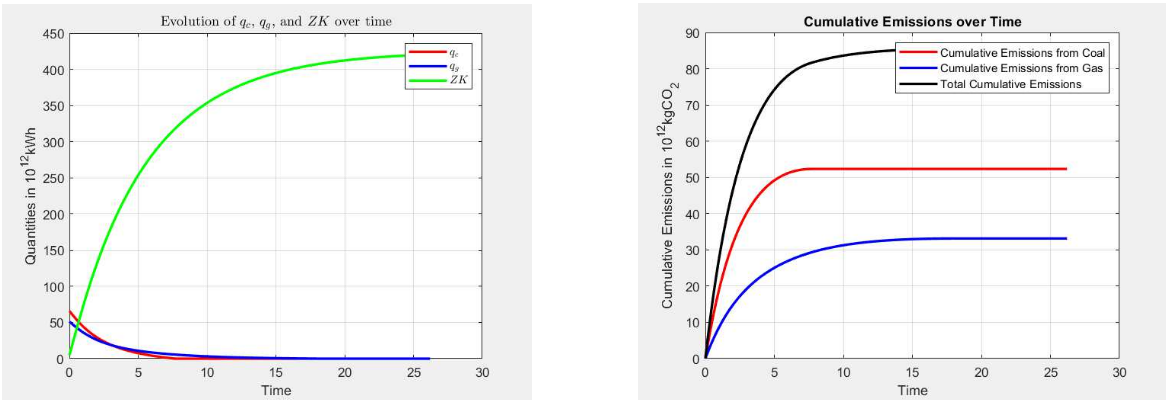
(a) Quantities of kWh supplied over time
(b) Cumulative emissions over time

Figure 9: Model with a subsidy of s=0.01

The area under q_c (red line) is $153,046.2 \times 10^9$, and the area under q_g (blue line) is $107,502.3 \times 10^9$. This means that the quantities of q_c and q_g produced are $153,046.2 \times 10^9$ and $107,502.3 \times 10^9$ kWh, respectively.

Using these quantities, we calculate the long-term emissions without a subsidy as follows:
- For coal, we have $153,046.2 \times 10^9 kWh * 0.35 kgCO_2/kWh = 53,566.2 \times 10^9 kgCO_2$.
- For gas, we have $107,502.3 \times 10^9 kWh * 0.20 kgCO_2/kWh = 21,500.5 \times 10^9 kgCO_2$.
Together, this results in total emissions of $75,066.6 \times 10^9$ kg of CO_2.

s=0.02:



(a) Quantities of kWh supplied over time

(b) Cumulative emissions over time

Figure 10: Model with a subsidy of s=0.02

The area under q_c (red line) is $149,514.0 \times 10^9$, and the area under q_g (blue line) is $165,712.9 \times 10^9$. This means that the quantities of q_c and q_g produced are $149,514.0 \times 10^9$ and $165,712.9 \times 10^9$ kWh, respectively.

Using these quantities, we calculate the long-term emissions without a subsidy as follows:
- For coal, we have $149,514.0 \times 10^9 kWh * 0.35 kgCO_2/kWh = 2,329.9 \times 10^9 kgCO_2$.
- For gas, we have $165,712.9 \times 10^9 kWh * 0.20 kgCO_2/kWh = 33,142.6 \times 10^9 kgCO_2$.
Together, this results in total emissions of $85,472.5 \times 10^9$ kg of CO_2.

8.4 Future research

8.4.1 Shared Investments

To implement a shared investment, the extension can be set up as follows:

Investment I at time t is $I_t \geq I_{g,t} + I_{r,t}$, where $I_{g,t}, I_{r,t}$ are the investments in gas and renewables, respectively.

The share of investments depends on the previous investments: $I_{j,t} = I_t(1 - \omega_j)$, $\omega_j \in (0,1)$, $\omega_j = \frac{I_{j',t-1}}{I_{t-1}}$, $j \in \{g, r\}$, j' is the other energy source in each case.

With a subsidy s for gas, the investments for gas increase: $\bar{I}_g = I_g + s$ and $\bar{I}_t = \bar{I}_{g,t} + I_{r,t}$.

It follows for the investment share:

$$I_{j,t} = I_t(1 - \omega_j),\ \omega_j \in (0, 1),\ \omega_j = \frac{\bar{I}_{j',t-1}}{\bar{I}_{t-1}}$$

8.4.2 Scarce gas resources

The cost function for gas could then be written as

$$k_g = \frac{1}{2}\alpha_g q_g^2 + \eta_g q_g + (\bar{q}_g - q_g)^{-\phi}$$

Here, $\bar{q}_g$ is the initial stock of gas at time $t = 0$.